Downscaling extreme precipitation in Ireland using combined peak-over-threshold generalised Pareto distribution model of varying parameters
Yassin Z. Osman, Rowan Fealy and John C. Sweeney

ABSTRACT

The paper describes downscaling of extreme precipitation in Ireland using a probabilistic method. The method described uses a combined peak-over-threshold (POT) – generalised Pareto distribution (GPD) approach in which the scale parameter of the GPD is allowed to vary with a dominant climate forcing at the location of interest. The dominant climatic forcing is represented by predictors selected from large-scale climatic variables provided by the NCEP/NCAR data. Data from six rainfall stations are used in the study to build the models for each station. The extRemes software is used to build the models as it allows parameters of the fitted distribution to vary as a function of covariate(s). The developed models were tested for goodness-of-fit with the observed data, and model fit was found to be much improved when the scale parameter was assumed to vary with the selected covariates. Return level – return period relations are developed based on the models developed and four future time periods are simulated to investigate the effects of climate change on both precipitation magnitude and frequency. Based on the findings of this research, significant changes in precipitation extremes are projected for Ireland, which includes wetter winters and drier summers, especially in inland areas.

Key words | covariates, extreme precipitation downscale model, extRemes software, generalised Pareto distribution, peak-over-threshold

INTRODUCTION

Changes in the tails of climate distributions are likely to lead to more significant impacts than just a change in the mean of the distribution. For example, the occurrence of extreme precipitation events that exceed the natural buffering capacity of a river catchment can result in severe flooding. Projected changes in the climate system, if realised, are likely to lead to an increase in the magnitude and occurrence of such extreme events. In order to minimise any potential future impacts of these events, knowledge about how the frequency and occurrence of extreme events are likely to change, as a consequence of changes in the climate system, is central to developing robust adaptation strategies. Therefore, determination of the likely recurrence period of extreme events when conducting studies to assess the impacts of climate change is an important step. One of the primary tools used for simulating future spatial and temporal changes in climate variables is the general circulation model (GCM). The GCM usually generates outputs at a relatively coarse resolution (typical grid scale is \( \sim 300 \text{ km} \times 300 \text{ km} \)), whereas impact studies typically require information at a point scale (e.g. field, catchment scale). Therefore a downscaling tool to translate the outputs of a GCM to a finer resolution scale is normally needed.

Much attention has been devoted recently to the topic of downscaling. This is largely driven by the fact that GCMs are better able to model large-scale climate variables (e.g. atmospheric pressure) than climatic parameters that vary at sub-grid scale resolution (e.g. precipitation).
Consequently, numerous methods have been developed (e.g. Wilby & Dawson 2007) to downscale GCM outputs to local and regional scales including dynamic regional modelling (e.g. Dethloff et al. 1996), pattern scaling (e.g. Santer et al. 1999), delta change methods (e.g. Leander & Buishand 2007) and statistical downscaling (e.g. Semenov 2008). The latter has become widely used in climate change impact studies largely due to its ease of implementation and reduced computational requirements, compared with dynamical climate modelling. Statistical downscaling methods are based on establishing relationships between environmental surface variables (predictands) and large scale atmospheric circulation variables (predictors). These relationships are then applied to a corresponding suite of circulation variables simulated by a GCM model in order to generate future scenarios of local climate (Karl et al. 1990; von Storch et al. 1993). In development of statistical relationships (or transfer functions) between observed variables and potential atmospheric predictors, both linear (e.g. multiple regression) and non-linear (e.g. neural networks) approaches are widely used (Wilby et al. 1998). The derived parameters are fundamentally assumed to be stationary and time invariant. Although this assumption cannot be fully verified, Charles et al. (1999) found that the assumption of time invariance in predictor–predictand relations may be robust provided that the choice of predictors is sensible.

Modelling of extreme event statistics (i.e. magnitude and return period) within a deterministic modelling framework is problematic as deterministic models tend to underestimate future extreme values which exceed those used during calibration of the models. This underestimation is attributed to a lack of stationarity in the derived model parameters. Therefore, a model which addresses this shortcoming is needed. The use of covariates, to scale model parameters, has previously been considered in Ireland, but not in the context of downscaling. Khaliq & Cunnane (1996) modelled point rainfall occurrences with a modified Bartlett-Lewis rectangular model. They applied a six-parameter version of the model to fairly long hourly rainfall data recorded at Valentia and Shannon Airport in Ireland. Five different sets of statistics of rainfall data for each month, assuming stationarity within the month, were used to estimate the six parameters of the model by Rosenbrock (1960) optimisation technique. Stability and sensitivity for the obtained parameters to number and type of rainfall statistics in a set were examined and an optimum set and number was derived. No use was made for climate variables in this model. The conditional distributions of rainfall depth obtained from the model compared favourably with the historical ones. Another study, undertaken by Demissie (2004) in a study of the effects of climate change on rainfall characteristics, employed atmospheric circulation and moisture variables from both the NCEP/NCAR reanalysis data and the HadCM3 GCM to model rainfall properties at Shannon, Mullingar and Rosslare synoptic stations. Initially, a statistical downscaling model was developed using both multiple linear regression and neural network models to predict local mean rainfall at these stations. Using the cluster point process model, Demissie (2004) then developed a stochastic model for simulating future extreme rainfall events in these stations by conditioning the parameters of this conceptual rainfall model upon the statistically downscaled mean rainfall properties obtained earlier. Results from the model suggested an increase in rainfall magnitude and in dry spell durations and a decrease in frequencies of rainfall depth. Kiely (1999) also investigated the impacts of climate change on precipitation and stream flow. He analysed five decades of hourly precipitation (at eight sites) and daily streamflow at four rivers in Ireland. In part of his study, he associated the trend changes in rainfall and streamflow with changes in the North Atlantic Oscillation index (NAO) that occurred in the mid-1970s.

Although the probabilistic nature and seasonality of extreme rainfall have been acknowledged in those studies, none of the previous studies has explicitly conditioned or associated change in the parameters of extreme rainfall on a climate variable (a covariate) or over time. Therefore, the present study seeks to fill this gap.

Globally, previous studies of modelling extreme rainfall in which model parameters are allowed to change with time or climate variables (covariates), are found in the work of Katz (1999), Coles (2001) and Katz et al. (2002). Katz et al. (2002), based on an earlier work by Coles (2001), presented a methodology for statistical downscaling of extreme events through the incorporation of covariates into the extremal distribution. The developed methodology fits extremal distributions by maximum likelihood (ML), similar to
the situation with time dependent parameters, but unlike a
deterministic trend variable, a covariate is itself a random
variable. Therefore, by fitting the extremal distribution con-
ditional on the values assumed by the covariate, the problem
reduces to that of a time varying parameter. For instance,
given the value of a covariate \( y \), the conditional distri-
bution of the extremal series could be assumed to follow a
generalised extreme value distribution with location param-
eter \( \mu(y) \), scale parameter \( \sigma(y) \) and shape parameter
\( \gamma(y) \). A typical parameterisation would be the same as in
Equation (1):

\[
\begin{align*}
\mu(y) &= \mu_0 + \mu_1(y) \quad \text{changing location with } y \\
\ln \sigma(y) &= \sigma_0 + \sigma_1(y) \quad \text{changing scale with } y \\
\gamma(y) &= \gamma \quad \text{unchanging skewness with } y
\end{align*}
\]

(1)

More generally, the covariate \( y \) could actually be a vector
(i.e. consisting of one or more covariates, say \( y_1, y_2, \) etc.).

The main factor in obtaining a good extremal model
in any location depends on selection of appropriate
covariate(s) that might have dominant effects on the local/
regional scale variable on an annual or seasonal time
scale. One natural candidate to serve as a covariate for
hydrologic extremes is the El Nino-Southern Oscillation
phenomenon, the dominant mode in global climate vari-
ation on an annual time scale (e.g. Katz et al. 2002). It has
been associated with climate anomalies (such as droughts
or floods) across large regions of the world. Similarly, the
NAO, which is the dominant mode of wintertime atmos-
pheric variability in the North Atlantic, has significant
influence on climate variability in Western Europe, and
specifically Ireland as highlighted by Kiely (1999).

Similar to the case of traditional deterministic downscal-
ing, in which large-scale atmospheric variables at grid point
level are the field from which input variables of the down-
scaling models are selected, these large-scale atmospheric
variables may also have the same effects on the extremal dis-
tribution parameters. Consequently, the local/regional
extremal events could be affected by a change in the pattern
of the large-scale atmosphere-ocean circulation at the grid
point level corresponding to it. Therefore, the large-scale
atmospheric variables are considered here as local covari-
ates which affect extremal events (e.g. extreme rainfall).

The methodology proposed by Coles (2003) and Katz et al.
(2002) in downscaling extremal events are applied in an Irish
context in the present research. However, unlike these studies,
the extremal models presented here are seasonally based and
their associated covariates are selected from the large-scale
atmospheric variables, provided by GCM outputs, at a grid
point level corresponding to Ireland. The basic assumption
made here is that parameters of a seasonal extremal distri-
bution model at a location/region change as a function of
large-scale atmospheric variables at the grid point level,
since these variables incorporated the effects of NAO.

The paper is organised as follows: A description for the
methodology used in modelling extremal distributions and
the software used is given in the next section. The data
used in the study are then described. There follows an expla-
nation of how the study is conducted and the steps involved
in developing the models. Results are presented and dis-
cussed and the final section gives a summary and
concluding remarks about the study.

EXTREMAL DISTRIBUTION MODEL

The two main models used for extreme values are the
annual maximum, or block maxima model (BM), and the
peak-over-threshold (POT) model. The BM model uses a
series of extreme values formed by selecting the highest
value in a year or a block and then proceeds with fitting a
statistical distribution to this extracted series. The POT
model on the other hand uses all data above a threshold
to form a series of extreme values and then proceeds with
fitting a statistical distribution to this series. A rigorous
discussion of the merits and demerits of each model and the
appropriate statistical distribution to be used with each
one is given in Cunnane (1989), Coles (2001) and Palutikof
et al. (2005) and only that part relevant to the current
study is mentioned here.

In the present study, the extremal model used is based
on the Extreme Toolkit developed by Gilleland et al.
(2005). The modelling concept of POT is used to model
extreme values series of precipitation, as it contains more
information than the annual maximum one. Thresholds
used in extracting the POT series, as will be explained
later, are determined for each site using the 90th percentile
of data series as a guide for maximum extreme series. The appropriate distribution normally associated with such a model, as mentioned in Cunnane (1989), Coles (2001) and Palutikof et al. (2003), is any one drawn from the family of generalised Pareto distribution (GPD). The distribution function, \( F(X) \), of the GPD is given by:

\[
F(X) = 1 - \left[ 1 + \frac{\epsilon}{\sigma}(X - u) \right]^{-1/\epsilon}
\]

where, \( x \) is the random variable, \( x > u \); and \( \sigma \) is the scale parameter, \( \sigma > 0 \), with \( u \) = a threshold, \( \epsilon \) = shape parameter.

Depending on the value of the shape parameter, \( \epsilon \), the distribution can be classified as GPD type I, type II or exponential as follows:

(i) if \( \epsilon > 0 \), the distribution is GPD type I,
(ii) if \( \epsilon < 0 \), the distribution is GPD type II,
(iii) if \( \epsilon = 0 \), the distribution is an exponential distribution defined by:

\[
F(X) = 1 - e^{-\frac{|X - u|}{\sigma}}
\]

The return level–return period relation, \( (X_T - T) \), is given by:

\[
X_T = u + \frac{\sigma}{\epsilon}[(\lambda T)^{\epsilon} - 1]
\]

for GPD type I and type II, and

\[
X_T = u + \sigma(\lambda T)
\]

for exponential distribution, where, \( \lambda = m/n \), where \( m \) is the number of peak over threshold extremes; and \( n \) is the total number of years, \( T = \) return period (recurrence period) in years.

The covariate concept is based on associating a climate variable(s), considered to hugely affect precipitation in the named location, with one or all the distribution parameters. In the present study, similar to Katz et al. (2002), only the scale parameter is allowed to vary with the dominant covariates \((y_1, y_2, \text{etc.})\), while the shape parameter is kept constant. This is based on the assumption that the shape parameter, a characteristic for extreme precipitation distribution in a location, is assumed to remain constant in the current and future periods. Two functional relations for the parameter with covariates are sought here. These are (Gilleland et al. 2005):

\[
\ln(\sigma(y_1, y_2)) = \sigma_0 + \sigma_1 \cdot y_1 + \sigma_2 \cdot y_2 \quad \text{Logarithmic relation, and,}
\]

\[
\sigma(y_1, y_2) = \sigma_0 + \sigma_1 \cdot y_1 + \sigma_2 \cdot y_2 \quad \text{Identity relation}
\]

where, \( \sigma(y_1, y_2) \) is the new value of the scale parameter as function of the covariates, \( \sigma_0 \) is an intercept in the linear relation, and \( \sigma_1 \) and \( \sigma_2 \) are the slope or trend of the variation in directions of \( y_1 \) and \( y_2 \). In the present study, the identity relation was used to describe change in the scale parameter, since the covariates (as will be explained below) are selected using stepwise regression.

**Parameters estimation**

After determining a threshold and forming the POT series, parameters of the fitted GPD need to be estimated. One of the methods used in estimating the parameters of the model is the ML method. The log-likelihood function to be optimised, for \( \epsilon \neq 0 \), is defined as (Gilleland et al. 2005):

\[
l(\sigma, \epsilon) = -m \log \sigma - (1 + 1/\epsilon) \sum_{i=1}^{m} \log \left( 1 + \epsilon \left( \frac{X_i - u}{\sigma} \right) \right) 
\]

when \( \epsilon = 0 \) (i.e. for exponential distribution) the log-likelihood function is defined as

\[
l(\sigma) = -m \log \sigma - \frac{1}{\sigma} \sum_{i=1}^{m} (X_i - u)
\]

One advantage of the ML over other methods of parameters estimation is its adaptability to changes in model structures. This advantage allows incorporation of model parameters when they change as a function of the covariates. The above likelihood functions will, respectively, change to the following forms:

\[
l(\sigma_0, \sigma_1, \sigma_2, \epsilon) = -\sum_{i=1}^{m} \left\{ \log \sigma(y_1, y_2) \right\} - (1 + 1/\epsilon) \log \left( 1 + \epsilon \left( \frac{X_i - u}{\sigma(y_1, y_2)} \right) \right) 
\]

(iii) if \( \epsilon < 0 \), the distribution is GPD type II.
\[ l(\sigma_0, \sigma_1, \sigma_2) = -\sum_{i=1}^{m} \left\{ \log \sigma(y_1, y_2) - \frac{X_i - u}{\sigma(y_1, y_2)} \right\} \]  

(10)

As analytical maximisation of the log-likelihood function is not possible, numerical optimisation techniques are always used for this purpose. These are generally techniques devoted to the solution of non-linear equations, such as Newton–Raphson, Method of Scoring and BHHH method (Long 1997). The numerical optimisation techniques of Nelder–Mead and Broyden–Fletcher–Goldfarb–Shanno, as described in Henningsen & Toomet (2011), are employed by the ‘extRemes’ software used in this study.

Threshold selection

Selection of an appropriate threshold is always difficult and represents a point of weakness for a POT model over others. On one hand, a threshold must be set high enough so that only true peaks, with Poisson arrival rates (Palutikof et al. 2003), are selected. If this is not the case, the distribution of selected extremes will fail to converge to the GPD asymptote. On the other hand, the threshold must be set low enough to ensure that enough data are selected for satisfactory determination of the distribution parameters.

Accordingly, a number of procedures have been used to aid in selecting an appropriate threshold for the POT model at a site. Two of the used procedures are:

(i) Mean residual life graphs (Davison 1984): This is a plot of the mean excess over threshold as a function of threshold. For a GPD model, the graph should plot as a straight line, and the appropriate threshold value can be chosen by selecting the lowest value above which the graph is straight line.

(ii) Model parameter graphs (Coles 2001): This plots an estimate of each parameter as a function of threshold. For a GPD model, estimates of shape parameter should be approximately constant, while estimates of scale parameter should be linear, and the appropriate threshold value can be chosen by selecting the lowest value at which the graph is straight.

In the present study, the 90th percentile has been used as a guide for selecting the appropriate threshold for precipitation. Using one or a combination of the procedures described above, thresholds guides are refined to yield appropriate ones.

Model diagnostic tests

As the reason for fitting a statistical model to a set of data is to make conclusions about some aspect of the population of the observed data, such conclusions could be sensitive to the accuracy of the fitted model. Thus, it is necessary to check the model accuracy and goodness-of-fit by checking its agreement with the data that were actually used to estimate it (model descriptive ability) and also checking its ability to predict future values (model predictive ability). Four types of model diagnostics are used in the present study to visually check the goodness-of-fit (descriptive ability) of the GPD to model the extreme values series. These are:

(a) Probability plot, which is a comparison of an empirical (usually percentage rank) and the fitted distribution function in Equations (2) or (3). In the case of a perfect fit, the data would line up on the diagonal of the probability plots as will be shown below.

(b) Quantile plot, which is also a comparison of an empirical form for estimating the exceedance and the inverse of Equations (2) or (3). Any departure from linearity indicates model failure in perfectly fitting the data.

(c) Return period plot, which shows the return period in years against the return level from Equations (4) or (5). Confidence intervals can be added to the plot to increase its informativeness. Empirical estimates for the return levels are also added to the plot to be used as a model diagnostic. If the GPD model is suitable for the data, the model-based curve and empirical estimates should be in reasonable agreement.

(d) Density function plot, which is a comparison of the probability density function of a fitted model with the histogram of the POT data. This is a less informative diagnostic for the model as a histogram varies substantially with the choice of grouping intervals, which makes its use difficult and subjective.

For checking the GPD model predictive ability, the split sample test method is used. The observed POT extreme series is divided into a calibration sample (1961–1990)
and a validation sample (1991–2000). The distribution is fitted to the first sample using GPD without covariates and the estimated parameters are used to obtain the associated probability with which the observed POT in both samples has occurred. A second GPD fit with covariates is then performed, and the new estimated parameters are used in conjunction with the probability obtained from the first fit to simulate model output. Correlation between the observed and simulated POT series is then established. Coefficient of determination of is then used to check the model predictive ability (or efficiency) in both samples.

**Choice of preferred model**

When GPD parameters are considered function in the covariates, there would be a number of possible models to choose from. The basic principle on choosing between models is parsimony, i.e. obtaining the simplest model (with the smallest number of parameters) that explains as much variation in the data as possible. So in order to choose between model fits, a test known as the likelihood ratio test is used. The test proceeds as follows:

With two model fits $M_0$ and $M_1$, where $M_0$ is a subset of $M_1$, $M_0 \subset M_1$ (i.e. $M_0$ is the model without covariates and $M_1$ is the model with covariates), the deviance statistic is defined as:

$$ D = 2\{l_1(M_1) - l_0(M_0)\} $$

(11)

where, $l_0(M_0)$ and $l_1(M_1)$ are the maximised log-likelihoods under models $M_0$ and $M_1$, respectively. Large values of $D$ indicate that $M_1$ explains substantially more of the variation in the data than $M_0$; small values of $D$ suggest that the increase in model parameter size does not bring worthwhile improvements to the model capacity to explain the data. Therefore, help in knowing how large $D$ should be before preferring model $M_1$ over $M_0$ is provided by the asymptotic distribution of the deviance function (Coles 2001). The test of hypothesis is performed as follows:

Model $M_0$ is rejected by a test at the $\alpha$-level of significance if $D > c_\alpha$, where $c_\alpha$ is the $(1-\alpha)$ quantile of the $\chi^2$ distribution with $\nu$ degree of freedom where $\nu$ is equal to the difference in the number of estimated parameters.

**Extreme Toolkit (extRemes) software**

The computer software used to fit the GPD model to POT series, which allows the parameter to change as function of the covariates, is the extRemes version 1.62 (Gilleland et al. 2005). The software, written in R language and benefiting from Coles (2001) ‘S’ functions, is based on the concept of ML for estimating GPD parameters. The key advantage of the software is that it facilitates the fitting of statistical distribution with covariates using the ML method and has options for choosing an appropriate threshold for POT series. The Toolkit is specifically designed to facilitate the use of extreme values theory in applications oriented towards weather and climate problems that involve extremes.

**DATA**

Observed daily precipitation (Prec) data for the period 1961–2000, for six selected synoptic stations representing both coastal and inland parts of Ireland (Figure 1) are used in the present study. The precipitation data from Valentia (No. 305), Dublin Airport (No. 532), Belmullet
(No. 1034), Birr (No. 4919), Rosslare (No. 2615) and Malin Head (No. 545) stations are obtained from Met Éireann. Daily grid-point data for the atmospheric variables, for the surface and upper atmosphere, shown in Table 1, are taken from Wilby & Dawson (2007) and they consist of NCEP (National Centre for Environmental Prediction) re-analysis data (Kalnay et al. 1996). For demonstrating the methodology proposed in the present study, climatic variables from the emission scenario A2 of HadCM3, extracted for the period 1961–2000, have been used. The NCEP and HadCM3 grid-point data will serve as potential candidates of covariates for the seasonal extremal models developed.

**METHODS OF ANALYSIS**

In this section, the steps followed to build the seasonal extreme precipitation models using the combined POT-GPD approach are summarised in the following seven steps:

**First**, the daily observed precipitation together with the corresponding atmospheric variables obtained from NCEP and HadCM3 are arranged into four seasons. Winter is defined as months December, January and February (DJF); spring as months March, April and May (MAM); summer as months June, July and August (JJA); and autumn as months September, October and November (SON). The lead and lag of variables in Table 1 have created additional covariate time series. A suffix of _1 (_2) is added to the present variable coding to represent a lagged time series of the variable (e.g. P_F_1, P5_U_2), whereas a suffix of +1 (+2) is added to represent a leading time series of the variable (e.g. MSLP +1, SHUM +2).

**Second**, threshold guides for an extreme seasonal series (u) of Prec for each station are obtained using their 90th percentile as a guide to comply with extreme event indices defined in the STARDEX project (2003). The threshold is then refined while fitting the model and the optimum threshold in each case is taken when estimated values of model parameters stabilise. The base period for the calculation of the thresholds is the period 1961–1990. This 30-year period, defined by the World Meteorological Organisation as the 30-year normal period, is considered representative of the present day climate and encompasses a range of natural variability (IPCC 2001). Using the threshold, seasonal series of precipitation extreme values are extracted for each station together with their corresponding possible set of covariates.

**Third**, covariates selection exercises are run using stepwise regression between the extracted POT precipitation series and the possible set of covariates. Initially, a cross-correlation is conducted between all possible seasonal covariates, at each station. This helps in excluding covariates demonstrating a high degree of co-linearity. Then, covariates-extreme value correlations are obtained for each station using stepwise regression. This analysis, in combination with the cross-correlation, allow determination of which covariates are most strongly correlated with the precipitation, which in turn helps in making an adequate selection of covariates for use in downscaling and reduces the problem of multi-co-linearity. A t-test for significance of the correlation between the precipitations POT series and each covariate, obtained via the stepwise regression, is then run to help select the most dominant covariates.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Climatic variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable name</td>
<td>Variable code</td>
</tr>
<tr>
<td>Precipitation (mm)</td>
<td>Prec</td>
</tr>
<tr>
<td>Mean temperature over a day (°C)</td>
<td>TEMP</td>
</tr>
<tr>
<td>Mean sea level pressure (hPa)</td>
<td>MSLP</td>
</tr>
<tr>
<td>500 hPa geopotential height</td>
<td>P500</td>
</tr>
<tr>
<td>800 hPa geopotential height</td>
<td>P800</td>
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<tr>
<td>Near surface relative humidity</td>
<td>RHUM</td>
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<tr>
<td>Near surface specific humidity</td>
<td>SHUM</td>
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<tr>
<td>Geostrophic airflow velocity</td>
<td>P_F</td>
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<tr>
<td>Vorticity</td>
<td>P_Z</td>
</tr>
<tr>
<td>Zonal velocity component</td>
<td>P_U</td>
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<td>Meridional velocity component</td>
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<td>Geostrophic airflow velocity (500 hPa)</td>
<td>P5_F</td>
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<td>Vorticity (500 hPa)</td>
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<td>Geostrophic airflow velocity (800 hPa)</td>
<td>P8_F</td>
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<td>Meridional velocity component (800 hPa)</td>
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Table 2 | Seasonal precipitation models, statistics and parameters

<table>
<thead>
<tr>
<th>Seasonal Model</th>
<th>Threshold</th>
<th>m</th>
<th>λ</th>
<th>Covariates</th>
<th>Parameters (No Cov)</th>
<th>Parameters (With Covariates)</th>
<th>Coefficient of Determination ($R^2$)</th>
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<tr>
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<td></td>
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<td>$\lambda_0$</td>
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<td>P_V</td>
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<td>P_U</td>
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validation. Values of $R^2$ for each period and season are also shown in Table 2.

Fifth, climate change driven seasonal return level–return period relations for each station are developed by extracting corresponding values of covariates from scenario A2 of HadCM3. The extracted covariates are used to generate possible future values assumed by the scale parameter ($\sigma(y_1, y_2)$), using fitting parameters of model $M_1$ and Equation (6). The shape parameter is considered constant. For each possible value of the scale parameter a value for the return level $X_T$ for a range of return periods $T$ years is calculated using Equation (4). Return periods considered are 2, 3, 5, 7, 10, 20, 30, 50, 75 and 100 years. Values of $X_T$ from model $M_0$ (referred hereinafter as NOCLM) are also calculated for the same return periods using Equation (4) and model parameters values from fit $M_0$.

Sixth, the maximum, average and minimum values of the calculated $X_T$ in each seasonal downscaling model are then obtained for the baseline period 0 (CLM1961-1990), period 1 (CLM1991-2020), period 2 (CLM2021-2050), and period 3 (CLM2051-2080). The number of years in each period ($n$) is 30 years.

Finally, for each $X_T$ series in the periods above, the maximum value of the series is taken to represent a point in the effective $X_T$–$T$ relation in that period. The max ($X_T$) and min ($X_T$) points obtained are finally plotted against return period $T$ to yield the affective seasonal return level–return period curve for any of the considered periods at all stations.

ANALYSES AND DISCUSSION OF RESULTS

In this section, results obtained from this study are analysed and discussed. This comes in two main parts. The first part is devoted to analysing the goodness of fit of the POT-GPD as a downscaling approach for extreme values of precipitation at all stations and how incorporation of covariates improves model predictability. The second part concerns discussion of how the developed downscaling models could be used to drive an effective seasonal return level–return period relation, and the usefulness of using these effective relations in estimating quantiles of different frequency in a future time.

Development of POT-GPD seasonal models

Following the steps described in the Methods section, the extRemes software is employed to build extreme precipitation seasonal models. The combined POT-GPD approach is used to build 24 seasonal models (four models for each station) in which the scale parameter of the GPD is allowed to vary as a function of two selected covariates. Table 2 presents the results of estimated parameters, selected covariates and efficiency of all developed seasonal models.

Development of the Valenti Autumn model (Prec0305-Aut) is used here to demonstrate how the results in Table 2 are obtained. The autumn precipitation series in Valenti is correlated with a set of possible autumn climate variables derived from NCEP data. Using the 90th percentile of the autumn precipitation, which is 13 mm/day, as a guide, an initial series of POT values is extracted. The stepwise regression process revealed that the best candidates for precipitation covariate in the location were MSLP, P5_U_2, P_F and P5_V_2. The Pearson correlation of each candidate with the precipitation series was 0.203 for MSLP, −0.214 for P5_U_2, −0.171 for P_F and 0.257 for P5_V_2. The $t$ statistics calculated for these correlation coefficients were 3.582, −3.582, −2.856, and 4.532, respectively. The corresponding critical value of $t$ from statistical tables, for a 5% level of significance is 3.524. Accordingly, and based on this $t$-test, the two most dominant variables having effect on precipitation in the location, which could serve as covariates, are P5_U_2 and P5_V_2 (the zonal and meridional velocity components at level 500 hPa).

Similarly, for other seasonal precipitation models in Valenti, the appropriate covariates are found as P_V and P5_Z for the spring model, P_V_2 and P_Z_2 for the summer model, and P5_Z and P_V_2 for the winter model. It can be observed that all appropriate covariates for Valenti seasonal models are associated with the zonal and meridional velocity and vorticity at various levels for the coastal location. The physical interpretation of this is that extreme precipitation events at Valenti (for all seasons) are much influenced by the zonal and meridional velocity and thus can be considered part of any predicting model of precipitation in the location.
After the appropriate covariates are determined, refining for the threshold to use in the model is followed. A first fitting for the model is performed with the guided threshold. Following the second procedure in the ‘Threshold selection’ section, a plot of threshold values against model parameters, as shown in Figure 2, is prepared. Based on the plots in Figure 2, selection of a threshold value between 15 and 16 mm/day is deemed suitable for the Prec0305-Aut model. Thus a threshold of 16 mm/day is chosen for this model and a new POT series is extracted. This procedure is applied in all developed models.

The POT-GPD autumn model for Valentia (Prec0305-Aut) is then built in two steps. Firstly the data is fitted to GPD to form model $M_0$. Diagnostic plots provided by the software are used to check the descriptive ability of model $M_0$. Figure 3 shows the four diagnostic plots, described in the section on ‘Model diagnostic tests’ above, for Valentia Prec0305-Aut model. The probability plot in Figure 3 shows good agreement between model and empirical prediction of the probability, which indicates that GPD fits the extreme series very well. The quantile plot, on the other hand, shows good agreement for lower quantile values and slightly departs from a straight-line relation for the higher values. Similarly, in the return period plot, almost all quantiles estimated by the model fall within the 95% confidence intervals produced empirically. Although not considered a strong diagnostic tool, the density plot of the POT data at this station resembles the general shape of the GPD density function. Accordingly, it could be deduced that GPD fits the extreme precipitation at this station very well as judged by these plots and can be used for modelling its extreme values.

Secondly, the data is fitted to GPD to form the downscaling model $M_1$ (P5_U_2 and P5_V_2 are used as covariates for the scale parameter). Fitting of GPD with covariates will result in two more parameters for model $M_1$. As can be observed from the results in Table 2, there is a change in the shape parameter value between models $M_1$ and $M_0$ as the distribution shifted form GPD I to GPD II. Two diagnostic plots for model $M_1$ are shown in Figure 4. The probability plot in Figure 4 is slightly different from that in Figure 3; however, all points are arranged along a straight line. The quantile plot of Figure 4 is an improvement over
the previous one as almost all quantile values fall along the straight line including the higher ones.

Preference of model $M_1$ over $M_0$ is judged by conducting the test described in the section on ‘Choice of preferred model’ above. As $M_0$ is the base model and $M_1$ is a different version of the base model (with two more parameters), the $\chi^2$ test would have a degree of freedom of 2 (i.e. $\nu = 2$). So, for a level of significance $(\alpha)$ of 0.05, the corresponding value of $\chi^2_2$, from statistical tables, is 5.9915. The deviance $D$ of Equation (11) is evaluated as 22.8477. Accordingly, fit $M_0$ is rejected and fit $M_1$ is preferred over it, and hence it is adopted as the perfect downscaling model for this station.

Following the above steps, all seasonal models for the six stations are built and their particulars are presented in Table 2. In all stations, it is found that addition of covariates to the base model ($M_0$) brings more improvements to the model and model $M_1$ is always found to be the best POT-GPD seasonal model at the station. Addition of covariates to the base model ($M_0$) transforms the model from a stationary model to a non-stationary model ($M_1$) with varying parameters, which is the main cause for improvements in the model predictability.

The developed seasonal models’ predictability and efficiency are further checked here by evaluating the value of coefficient of determination, $R^2$, yielded by correlating the observed and simulated extreme series, as explained in the ‘Methods’ section. For a significance level of 0.05, values of coefficient of determination are found to be very high (more than 80%) for all models for both the calibration and validation periods, as shown in the last two columns of Table 2. This, beside the likelihood ratio test, reinforces the postulation made in this study that downscaling of extreme precipitation is better addressed under non-stationary extreme value theory. This entails choosing the right extremal model and an efficient parameter estimation method that is compatible with it. The best outcome of this combination has been reflected in the extremal model chosen here, the POT-GPD-ML.

Visual comparison for the degree of agreement between the observed and simulated extremes yielded by the developed seasonal models of Table 2 are shown in Figures 5(a)–5(d) for Valentia station and in Figures 6(a)–6(d) for Birr station. In all of these figures, the large (+) sign represents a dividing point between the calibration and validation periods.
periods used in evaluating the coefficient of determination. The perfect matching between the observed and simulated series in these figures is a clear indication of the correct choice of the statistical distribution, appropriate covariates, and the likelihood technique used in estimating its parameters.

**Climate driven return level \( (X_T) \) – return period \( (T) \) relations**

The \( X_T-T \) relations discussed here are developed using the seasonal downscaling models built for each station. Graphical forms of this relation are shown here for the purposes of investigating how climate change can possibly affect the magnitude and frequency of future extreme precipitation. Relations for Valentia and Birr stations are used here for demonstration.

Figures 7(a)–7(d) and Figures 9(a)–9(d) show graphs of Valentia and Birr effective precipitation return levels–return period relations. Each seasonal relation at a station shows five curves; one for each modelling period described in the Methods section, and a fifth curve to represent the relation from NOCLM \( (M_0) \). Figures 7(a)–7(d) for Valentia station demonstrates that climate change has a major effect in generally increasing return level/quantile magnitude as dictated by the upward shift of all climate-driven relation curves from the NOCLM \( (M_0) \) curve. For example, for a return period of 100 years for the Valentia autumn return level–return period relation, NOCLM \( (M_0) \) predicts a quantile magnitude of 120 mm/day, the baseline period predicts it as 139, period 1 predicts it as 143, period 2 predicts it as 151, and period 3 predicts it as 135 mm/day. So, on average, climate change is likely to increase the quantile magnitude by about 20% at this location. This finding is similar to the recommendations made in the Flood Estimation Handbook (FEH 1999), which adopted an addition of 20% to any estimated flood magnitude to cater for future climate change. Within the Valentia seasonal relations, e.g. in the summer season, the baseline period curve predicts higher quantile magnitude than the second period curve (indicating a decrease in precipitation in summer with climate change), whereas for spring and winter seasons, all periods’ curves yield almost equal increases in quantile magnitude.
The effect of climate change on return period is further explained by curves of the autumn season for Valentia shown in Figure 8. In this figure, the return periods for a precipitation of 120 mm/day are found as 100 years for $M_0$ (NOCLM), 35 years for period 3, 27 years for the baseline period, 22 years for period 1, and 17 years for period 2. This means that due to climate change the frequency of 100 years precipitation is reduced to 35 years or less. Thus, based on these results, wetter conditions are expected with the current pattern of climate change at this location. The above demonstration of the significant effects of climate change on extreme magnitude and frequency explains why it is necessary to take this effect into consideration when planning or designing for the future in the natural environment. Consequently, the effective climate-driven relations developed in this study can be very useful in this respect, especially when making plans at catchment levels.

Similar results of an increase in extreme magnitude of precipitation can also be noticed from the seasonal relations of Birr station, which are shown in Figures 9(a)–9(d). The curves in these figures show the same pattern as those of Valentia; however the percentage increase brought about by the climate change effect is somewhat different. Here, the percentage increase in the quantile magnitude, for example for a 100 years return period, varies within the seasons between 20 and 50%, and the frequency of occurrence of such a quantile is much shorter than those of Valentia. This could be attributed to the greater influence of climate change at this location than in Valentia.

**SUMMARY AND CONCLUSIONS**

Development of seasonal downscaling models for extreme precipitation, within a probabilistic non-stationary framework, has been addressed in this research. The objective is to develop a downscale model which is capable of maintaining extreme value characteristics (magnitude and frequency) in addition to ability to predict climate change effects on these characteristics. The seasonal models developed here are mainly based on fitting GPD to the observed extreme values of precipitation, which formed by using the peak over threshold model. The main theory behind choosing this modelling approach is that parameters of the fitted distribution vary as a function of dominant climatic variable(s) in the area known as covariate(s). These covariates are
Figure 8 | Return period of precipitation of same magnitude from different Prec0305-Aut models.

Figure 9 | Return level versus return period plot (a) Prec4919-Aut; (b) Prec4919-Spr; (c) Prec4919-Sum; and (d) Prec4919-Win.
selected here from large-scale atmospheric circulation variables, at grid point level, provided by GCM models. In this study, only the scale parameter of the GPD is allowed to vary as a function of the dominant covariates in the location. The methodology is demonstrated by using observed precipitation data from six stations, representing coastal and inland parts of Ireland and covariates derived from climatic variables provided by scenario A2 of HadCM3. A total of 24 (4 for each stations) seasonal downscaling models were developed in the study using the extReme software. Effective climate-driven return level–return period relations are also derived at each station based on the models developed. Results are presented then analysed and discussed. Concluding remarks are summarised as follows:

- The combined seasonal POT-GPD models developed here are proved to model the extreme behaviour of precipitation in a very successful manner. The dominant covariates obtained were found to associate with the physical cause of precipitation in the location.
- Demonstration results of the models’ descriptive and predictive abilities have reinforced the idea that modelling of extreme values is better addressed within the probabilistic non-stationary framework of modelling.
- All the developed seasonal downscaling models are demonstrated to be well representative of the future situation under climate change. Taking uncertainties into consideration, the developed models could be suitable for use to downscale precipitation quantiles for a given return period for planning purposes. All that is needed is to follow the steps mentioned in the Methods section to build these models.
- Precipitation climate-driven return level–return period relations derived here suggest that there is a possible increase in extreme precipitation magnitude and frequency in Ireland with the current and future enforcing of climate change; the influence of climate change has been much observed in inland parts of Ireland. This means that wetter conditions are expected with current and future climate change unfolding. The expected percentage increase in precipitation magnitude is around 20%.
- The expected increase in extreme conditions of precipitation would have adverse effects on the natural environment and socio-economic activities. Therefore the models and effective quantile return period relations developed here could be used at the planning stage of environmental projects or for water resources management and agricultural activities.

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REFERENCES


Kiely, G. 1999 Climate change in Ireland from precipitation and stream flow observations. Advances in Water Resources 23, 141–151.


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