DEMAND UNCERTAINTY, R&D LEADERSHIP AND RESEARCH JOINT VENTUREs

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Abstract

This paper analyses the desirability of RJV formation when firms may choose their R&D investment before or after any demand uncertainty is resolved. If a R&D leader accommodates a follower, multiple Nash equilibria are possible under both R&D competition and RJV formation. If a R&D leader prevents activity by the follower, this is only expected to be profitable at very low spillover and unit R&D cost levels. Whether R&D leadership when competing in R&D is expected to be more profitable than waiting and forming a RJV will depend on unit R&D costs and spillovers. Maximising expected welfare may require an active role for government.

JEL Classification: D21, D81, L13

# Thanks to Dermot Leahy and J. Peter Neary for helpful comments and suggestions and to seminar participants at NUI Maynooth and the Irish Economic Association Annual Conference. Any remaining errors are solely those of the author.

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1. Introduction

As a result of recent challenging economic circumstances, many individual and regional economies have emphasised the importance of indigenous industry ‘moving up the value chain’ or entering the ‘smart economy’ by producing goods and services requiring higher skills and a greater ability to absorb technological advances developed elsewhere. The Lisbon Strategy and the Europe 2020 Strategy, for example, aim to encourage innovation in the EU, while Joint Ventures can benefit from a ‘block exemption’ from EU competition law.

Innovation can suffer from the public good problem as knowledge of a firm’s innovation processes and results may ‘spill over’ to its rivals, reducing the private incentive to undertake research and development (R&D), as no innovator can fully appropriate the benefits of its innovation. Other problems are that the outcome of any innovation process may be uncertain and/or the financial investment required may be relatively large. Also, innovating firms may duplicate the innovation efforts of its rivals, which is socially wasteful.

Classic examples of public solutions to the private innovation problem are the introduction of patents and R&D subsidies. One private solution is for innovators to co-operate in R&D by forming a Research Joint Venture (RJV), where innovation is undertaken to maximise the sum of joint profits of all RJV members. Also, innovators can decide on the level of information sharing within the RJV, so that effective spillover levels become endogenous, and can also co-ordinate their innovation efforts to avoid any duplication.

One possible problem is that co-operation at the innovation stage may be extended, implicitly, to the final output market, increasing the market power of the RJV members. Another issue is that RJV’s may be formed by a subset of firms to attain, or increase, a competitive advantage over other firms, possibly inducing the exit of non-RJV firms and increasing the market power of RJV firms. RJV’s may also be formed to prevent entry into an industry in order to maintain or increase existing levels of market power that would be reduced in the absence of RJV formation. This may be beneficial in terms of greater consumer welfare if increased innovation reduces marginal production costs and, consequently, final output prices. On the other hand, this may be a temporary outcome as prices may increase if non-RJV firms are forced to exit the industry or potential entrants are faced with greater barriers to entry. In looking at the desirability of RJV formation, therefore, any regulatory authority must compare any possible pro and anti-competitive effects.

While firms can undertake R&D in order to improve the quality of their product (product innovation) or reduce marginal production costs (process innovation), this paper focuses on the latter. In general, the existing literature is favourable towards such ventures, as they can lead to greater incentives to undertake R&D, greater total industry profits and, possibly, greater welfare. A seminal contribution to the literature on RJV formation in the
presence of exogenous R&D spillovers is D’Aspremont and Jacquemin (1988), where firms can compete in R&D, by choosing their R&D levels to maximise own profits, or co-operate in R&D (form a RJV), where each firm chooses its R&D to maximise the sum of joint profits. Irrespective of what happens at the R&D stage, firms cannot co-operate in the output market, in accordance with antitrust regulations.

From the firms' perspective, RJV formation is weakly preferred to R&D competition. From a welfare perspective, the desirability of RJV formation depends on whether output is exported in its entirety or consumed domestically. In the former, RJV formation is weakly welfare-enhancing for all spillovers. In the latter, RJV formation should only be encouraged if spillover levels are relatively high. At relatively low spillovers, higher R&D investment under R&D competition leads to higher consumer welfare and this dominates the higher profits under RJV formation. Conversely, when spillovers are relatively high, RJV formation induces lower prices and higher profits so that welfare is higher.

An important point to note in the D’Aspremont and Jacquemin model is that even in the case of RJV formation, effective spillovers remain exogenous. Poyago-Theotoky (1999) showed that firms will wish to fully share information when forming a RJV. Kamien, Muller and Zang (1992) conclude that RJV cartelisation is most desirable as R&D investment and profits are highest, while output prices are lowest. One characteristic that is shared by all of these papers is that firms simultaneously undertake their R&D investment.

A number of papers examine the empirical evidence regarding RJV formation. Among these, Hernan, Marin and Siotis (2003) look at European data and find that industry concentration, firm size, technological spillovers and R&D intensity increase the likelihood of forming a RJV while patent effectiveness reduces it, concluding that “…knowledge diffusion is central to our understanding of RJV formation”. Roller, Tombak and Siebert (2007) analyse US data and find that firms that are similar in size, have already participated in other RJV’s and produce complementary products are more likely to form RJV’s.

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2 RJV profits are always greater, except for spillovers of ½ when the two cases are identical.

3 As there are no consumer welfare considerations, welfare is measured by total industry profits. This point was noted by, among others, Neary and O’Sullivan (1999).

4 A similar point was made by Motta (1992) in the context of a vertical product differentiation model where firms engage in product innovation. In this case, the threshold spillover was not identical due to a different demand specification.

5 KMZ define a RJV as a situation where firms fully share information between themselves, while cartelisation refers to when the firms choose their R&D investment in order to maximise joint profits.


7 The authors also provide theoretical evidence that large firms will not form RJV’s with smaller firms.
The other relevant theoretical literature strand is that of market leadership in the presence of uncertain demand. Several papers have analysed the effects of a firm having a first-mover advantage, either as an incumbent monopolist (e.g. Dixit (1980)), or as a duopolist (e.g. Spencer & Brander (1992)). Many of these papers focus on the case of output leadership, but also extend their models to ‘capacity’ leadership, where capacity can be interpreted as, for example, production facilities, advertising or R&D investment.

Spencer and Brander (1992) looks at where one firm in a duopoly has an exogenously given first-mover opportunity and constant marginal production costs depend on a firm’s ‘capacity’ level. The issue facing a ‘leader’ was whether to ‘commit’ to choosing its output or capacity before any uncertainty is resolved, or to remain ‘flexible’ and choose it after the true level of demand is realised. A leader ‘commits’ if demand volatility is sufficiently low and remains flexible otherwise. On the other hand, if both firms could choose to commit, the Nash equilibrium is where both firms either commit or remain flexible, with the equilibrium outcome depending on the volatility of the demand shock. For a range of variance values, there are multiple Nash equilibria.

Spencer and Brander expand the Dixit framework to the case of uncertainty about the ‘follower’s’ costs, where the firms’ marginal production costs are increasing in output. This framework allowed for entry to be unprofitable under a number of different circumstances. Entry is ‘blockaded’ if it is never profitable even if both firms are flexible and the demand shock is at its maximum. Entry is ‘actively prevented’ if it is unprofitable when the leader commits some capital, but would be profitable for the maximum value of the demand shock if both firms are flexible. Another possibility is that entry is profitable for the lowest value of the shock when the leader commits its capital, so that the leader may ‘manipulate’ entry by over-investing in capital when committing or by remaining flexible. Finally, the leader may engage in ‘probabilistic entry deterrence’ whereby the higher its committed capital, the more likely it will be that entry is deterred as the greater must be the realised value of any shock to make entry profitable. Though the Spencer and Brander paper is useful in looking at firms’ incentives in the face of uncertainty, there are no spillovers in ‘capacity’ choice, the firms’ marginal production costs are increasing in output and linear in capacity costs, while the firms never ‘co-operate’ in capacity investment if they remain flexible.

Rossell and Walker (1999), using the D’Aspremont & Jacquemin framework, look at the effect of incumbent R&D leadership in the presence of potential entry when there are R&D spillovers. While noting the standard outcomes of blockaded entry, entry accommodation and entry deterrence, they also argue that for relatively high spillovers, the incumbent may, depending on fixed entry costs, choose its R&D in order to solicit entry into the industry. In particular, the higher are fixed costs, the more likely it is that entry will be blockaded, though the incumbent will prefer accommodation to benefit from relatively high spillovers. In this
case, the incumbent may choose some R&D level that makes entry profitable. The lower are fixed costs, the more likely it is that the leader will choose to accommodate the entrant. In this paper, however, there is no co-operative behaviour, nor is there any welfare analysis.

Maggi (1996) finds that even if firms are ex-ante identical and have identical entry opportunities, an asymmetric equilibrium occurs where one firm chooses to commit to capital investment while the other remains flexible. Again, there are no spillovers in capacity investment, nor do firms co-operate in capacity choice.

Dewit and Leahy (2006) extends the literature further and allows for a lag between when firms commit to investing in capital and when such investment actually occurs. The situation where firms simultaneously commit to, and invest in, their capital investment is referred to as ‘action commitment’, with the case where firms choose the timing of their investment, but not the capital level, denoted as ‘observable delay’. For the purposes of this paper, it is the former case that is relevant.

In the ‘action commitment’ case, the equilibrium outcome depends on the effectiveness of R&D (related to the inverse of unit R&D costs) and the volatility of the demand shock. Depending on these values, the firms may either invest simultaneously or one firm is a leader. In some cases, there may be multiple equilibria. The lower the volatility of demand, the more likely it is that both firms will commit to investing in capital. At relatively high volatility levels, both remaining flexible becomes more likely to be the equilibrium outcome. At intermediate levels, a sequential investment equilibrium is likely. As with Spencer & Brander, however, Dewit and Leahy assume that there are no spillovers in capacity choice and there is no co-operative behaviour by the firms.

This paper, therefore, seeks to bring together a number of strands in the industrial organisation literature by complementing some of the papers referred to above in order to attain a greater insight into the incentives for firms to form Research Joint Ventures. While the incentive to form a RJV when firms simultaneously choose their R&D levels under certain demand conditions is well-known, this paper seeks to determine whether, under uncertain demand conditions, a R&D ‘leader’ will wish to form a RJV with a ‘follower’. In particular, issues of RJV formation, strategic entry accommodation, strategic entry deterrence and investment under uncertainty are analysed.

The structure of this paper is as follows. Section 2 describes the model. Section 3 looks at where both firms either ‘jump’ or ‘wait’ and simultaneously undertake their R&D investment. In Section 4, R&D levels are chosen sequentially but the ‘leader’ accommodates the follower. In Section 5, the simultaneous and sequential R&D games of Sections 3 & 4 are compared. Section 6 looks at where a ‘leader’ now seeks to prevent activity by the follower.
either for certain or in expectation. Section 7 compares the cases of accommodation and activity prevention by simulating the models of the previous sections. Section 8 concludes.

2. The Model

This paper looks at the incentives of risk-neutral profit maximising firms to form a RJV when the firms may sequentially undertake their R&D investments due to uncertainty about the true level of demand. The uncertainty is assumed to be resolved before the firms simultaneously choose their profit maximising output levels. The issue facing each firm is whether to ‘wait’ and delay its R&D investment until after the uncertainty is resolved, or to ‘jump’ and invest before the true demand level is known and possibly be a R&D leader. The interesting question is whether a ‘leader’ will wish to form a RJV with a ‘follower’. Alternatively, a ‘leader’ may prevent a ‘follower’ from becoming active in the industry.

The model is a one-shot game where two symmetric output-setting firms that produce an homogenous good undertake process R&D. Each firm incurs the direct cost of R&D investment and also receives an exogenous R&D spillover from its rival, the effect of which is to reduce own marginal production costs. To comply with antitrust regulations, the firms always remain rivals in the output market. All production takes place in one economy, while output may be consumed domestically or exported in its entirety to another economy.

It is assumed that both firms have costlessly entered the industry by, say, obtaining a licence from the government or a regulatory authority. If one firms ‘jumps’ and becomes a leader, the issue is whether, through its R&D choice, it will accommodate the follower or prevent it from becoming active in the industry. 8, 9 If the leader accommodates the follower, the firms may either compete in R&D, by choosing their R&D investment to maximise own profits, or co-operate in R&D (form RJV) by investing in R&D to maximise joint profits. Alternatively, if the leader seeks to prevent the follower from becoming ‘active’, there are two possible cases. Firstly, certain activity prevention where a leader chooses its R&D to ensure that the producing output is never profitable for the follower for any level of demand and the leader is always an output market monopolist. On the other hand, expected activity prevention is where a leader chooses its R&D to ensure that activity is expected to be unprofitable for the follower, given the expected value of demand, so that the leader may be an output market monopolist depending on the realised value of demand. In both cases, the leader competes in

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8 Intuitively, the latter is equivalent to entry deterrence in the absence of fixed costs. This paper’s ‘activity prevention’ differs from Spencer & Brander’s ‘actively prevented entry’ where entry was unprofitable if the leader committed some capital, but profitable for the highest value of the demand shock when both firms were flexible.

9 While the main focus of this paper is on RJV formation, the issue of activity prevention is analysed given the possible sequential move order.
R&D with the follower. If both firms ‘jump’ or ‘wait’, it is assumed that R&D levels are chosen simultaneously, while if a firm decides to ‘wait’, it may be a follower in R&D.

When firms simultaneously choose their R&D levels, there are four stages to the game. If both firms ‘wait’, then in the first stage, the demand uncertainty is resolved. Secondly, the firms decide whether or not to commit to forming a RJV. In the third stage, the firms choose their R&D levels, while in the final stage the firms choose their output levels. On the other hand, if both firms ‘jump’, the demand uncertainty is resolved after the firms choose their R&D levels, but before the output stage.

If the firms sequentially choose their R&D levels, the number of stages will depend on whether the leader accommodates the follower or prevents it from becoming active, either for certain or in expectation. Under accommodation, there are five stages. In the first stage, the firms decide whether or not to form a RJV. Secondly, the leader chooses its R&D. The demand uncertainty is then resolved followed by the follower choosing its R&D level. Finally, the firms choose their output levels.

If, on the other hand, the leader seeks to prevent the follower from becoming active, the number of stages depends on whether the leader does so for certain or in expectation. In the former case, there are three stages with the leader’s R&D choice in the first stage being followed by the true demand level being revealed. Finally, as the follower decides not to become active, the leader chooses its monopoly output level. In the latter case, there may be three or four stages depending on the level of demand. The first two stages are as in the certain prevention case. In the third stage, if activity is profitable, the follower chooses its R&D level and this is followed by both firms choosing their output levels. If, however, activity is not profitable, the leader chooses its monopoly output level in the third stage.

The firms are assumed to face the linear inverse demand function

\[ p(Q,u) = a - bQ + u \]  

where \( Q = q + q^* \) is total industry output and \( u \in [\bar{u}, \tilde{u}] \) is a uniformly distributed demand shock with an expected value of zero and a variance of \( \sigma^2 \).\textsuperscript{10,11} Given \( E(u) = 0 \), the variance of the demand shock is

\[ \sigma^2 = E(u^2) = \int_{-\bar{u}}^{\tilde{u}} u^2 f(u) du = \frac{\bar{u}^2}{3}. \]

Marginal production costs are a linear function of own and rival R&D and are

\[ c(x,x^*) = A - \theta(x + \beta x^*) \geq 0 \quad \text{and} \quad c^*(x,x^*) = A - \theta^*(\beta^* x + x^*) \geq 0 \]  

for the respective firms, where \( \theta, \theta^* \geq 0 \) denote the effectiveness of R&D in reducing marginal production costs and \( 0 \leq \beta, \beta^* \leq 1 \) are exogenous R&D spillover parameters. Given this, \( \theta \beta \) and \( \theta^* \beta^* \) are the effective spillovers of the firms. It is assumed that \( 0 < A < a - \bar{u} \) to

\textsuperscript{10} This is identical to the set-up of Spencer and Brander (1992) and Dewit and Leahy (2006).

\textsuperscript{11} In what follows, the non-representative firm is denoted by *.
ensure positive outputs, irrespective of the magnitude of the demand shock. R&D costs are convex in R&D, thereby exhibiting diminishing returns to R&D, and are given by $\Gamma(x) = \gamma x^2/2$ and $\Gamma(x^*) = \gamma x^*^2/2$ for the respective firms, where $\gamma > 0$ is a measure of unit R&D costs.

The profit of the representative firm is

$$\pi(q, q^*) = (p(Q, u) - c)q - \Gamma(x) = [a - bq - bq^* + u - (A - x - \beta^*)]q - \frac{\gamma x^2}{2}$$

(3)

Welfare is defined to be the sum of consumer surplus and total industry profits:

$$W(Q) = \frac{bQ^2}{2} + \pi + \pi^*$$

(4)

where the first term on the right hand side of (4) is a measure of consumer surplus from (1).

In what follows, $N$ denotes R&D competition, $C$ denotes RJV formation (R&D co-operation) and $D$ denotes activity prevention.\(^\text{12}\) Also, $U$ refers to the stage at which the uncertainty is resolved, $R$ denotes an R&D stage and $O$ denotes the output stage. These letters will denote the various games played by the firms. For example, RUROC is the case where the ‘leader’s’ R&D ($R$) occurs before the uncertainty ($U$) is resolved, after which the follower’s R&D ($R$) is then chosen before the output stage ($O$) while the firms form a RJV ($C$).\(^\text{13}\)

### 3. Simultaneous R&D Investment

There are two possible scenarios in which the firms simultaneously choose their R&D levels. Both firms can ‘wait’ until the uncertainty is resolved or ‘jump’ and undertake their R&D before the true demand level is known. Irrespective of what the firms do, the output stage is unchanged as the true demand is known and the firms always compete in output.

Maximising (3) with respect to the representative firm’s output, and doing likewise for the other firm, implies that ex-post output levels are given by

$$\left[\begin{array}{c} q \\ q^* \end{array}\right] = \frac{1}{3b} \left[\begin{array}{c} \alpha + u + (2\theta - \theta^* \beta^*)x + (2\theta\beta - \theta^*)x^* \\ \alpha + u + (2\theta^* \beta^* - \theta) x + (2\theta^* - \theta\beta) x^* \end{array}\right]$$

(5)

where $\alpha = a - A$.

#### 3.1 Post-uncertainty investment (both ‘wait’) (URON & UROC)

#### 3.1.1 R&D competition (URON)

\(^\text{12}\) It is assumed that accommodation is the default case, so that prevention/deterrence is denoted by D.

\(^\text{13}\) Similarly, URON refers to where firms compete in R&D and ‘wait’ until the uncertainty is resolved, while RUOC is where the firms form a RJV and undertake their R&D before the demand shock occurs.
Each firm chooses its R&D in order to maximise its own profits so that the representative firm’s first-order R&D condition is

\[
\frac{d\pi}{dx} = \frac{\partial \pi}{\partial \alpha x} + \frac{\partial \pi}{\partial q^*} \frac{\partial q^*}{\partial x} = 0
\]  

(6)

The direct effect of R&D is the marginal benefit of R&D, the reduction in variable production costs, less the marginal R&D cost. As the firms are output market rivals, there may be strategic considerations to each firm’s R&D investment. Using (1), (2), (3) and (5) in (6), the representative firm over (under) invests in R&D when \( \theta \beta^* < (>) \theta/2 \).\(^{14}\) If the effective spillover to its rival is sufficiently low, each firm over-invests in R&D to profit-shift from its rival as R&D is a strategic substitute. Conversely, if spillovers are sufficiently high, R&D is a strategic complement and each firm ‘free-rides’ on its rival’s R&D by under-investing in R&D.\(^{15}\) Ex-post symmetry implies that \( \theta = \theta^* \) and \( \beta = \beta^* \) and the firms over (under) invest in R&D when \( \beta < (>) \frac{1}{2}. \)\(^{16}\)

Given (5) and (6), profit maximising R&D levels are

\[
x^N = x_0^N = \frac{2\theta(\alpha + u)(2 - \beta)}{9b\gamma - 2\theta^2(2 - \beta)(1 + \beta)}
\]  

(7)

that decrease in the effective spillover and increase in the demand shock.\(^{17}\) As spillovers increase, a firm increasingly benefits from its rival’s R&D and this reduces each firm’s incentive to undertake R&D. Given \( E(u) = 0 \), the expected R&D of the firms is

\[
E(x^N) = E(x_0^N) = \frac{2\alpha \theta(2 - \beta)}{9b\gamma - 2\theta^2(2 - \beta)(1 + \beta)}
\]  

(8)

Ex-post profit maximising R&D levels are greater (lower) than expected levels when the actual shock is positive (negative). As the expected marginal private return to R&D per unit of output is identical to the actual return, the firms expect to over (under) invest in R&D when \( \beta < (>) \frac{1}{2}. \) Substituting (7) into (5) to solve for output levels, using these in (3) and given \( E(u^2) = \sigma^2 \), expected profits are

\[
E(\pi^N) = E(\pi_0^N) = \frac{\gamma(\alpha^2 + \sigma^2)[b\gamma - 2\theta^2(2 - \beta)(1 + \beta)]^2}{[b\gamma - 2\theta^2(2 - \beta)(1 + \beta)]^2} > 0
\]  

(9)

which are positively related to the variance of the demand shock,\(^{18}\) while expected welfare is

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\(^{14}\) Over (under) investment occurs when, in the first stage, firms undertake R&D investment where the marginal private benefit of R&D is less (greater) than the marginal private cost. This may not be profit maximising at the R&D stage but leads to higher output market profits in the second stage, thereby increasing total profit.

\(^{15}\) A firm’s R&D is a strategic substitute (complement) for its rival’s R&D when an increase in its R&D investment reduces (increases) the marginal profitability of its rival’s R&D. If R&D is a strategic substitute (complement), R&D reaction functions are downward (upward) sloping in R&D space.

\(^{16}\) This is consistent with D’Aspremont & Jacquemin where \( \theta = \theta^* = 1 \).

\(^{17}\) Each firm’s second-order R&D condition requires \( 9b\gamma - 2\theta^2(2 - \beta)^2 > 0 \). To ensure analysis for all spillover levels, this implies that by \( \gamma \) is large enough to satisfy \( 9b\gamma - 2\theta^2(2 - \beta)(1 + \beta) > 0 \).
\[ E(W^N) = \frac{2\gamma(\alpha^2 + \sigma^2)[18b\gamma - 2\theta^2(2 - \beta)^2]}{9b\gamma - 2\theta^2(2 - \beta)(1 + \beta)} \] (10)

which is also positively related to the volatility of the demand shock.

### 3.1.2 RJV formation (R&D co-operation) (UROC)

The firms now choose their R&D to maximise the sum of joint profits so that the representative firm’s first-order R&D condition is now

\[
\frac{d(\pi + \pi^*)}{dx} = \left[ \frac{\partial \pi}{\partial x} + \frac{\partial \pi^*}{\partial q^*} \cdot \frac{\partial q^*}{\partial x} \right] + \left[ \frac{\partial \pi^*}{\partial q} \cdot \frac{\partial q}{\partial x} \right] = 0
\] (11)

The direct marginal benefit of R&D is now the reduction in variable production costs of all RJV members. Using (1), (2), (3) and (5) in (11), the representative firm over (under) invests in R&D if \( \theta^* \beta^* < (>) - \theta \), so that the firm under-invests in R&D for any R&D spillover.\(^{19}\) Ex-post symmetry implies \( \theta = \theta^* \) and \( \beta = \beta^* \) so that, given (5) and (11), R&D levels are

\[
x^C = x^C = \frac{2\theta(\alpha + u)(1 + \beta)}{9b\gamma - 2\theta^2(1 + \beta)^2}
\] (12)

where R&D is now increasing in the effective spillover, given strategic complementarity of R&D.\(^{20}\) Given \( E(u) = 0 \), expected R&D levels are

\[
E(x^C) = E(x^C) = \frac{2\theta(1 + \beta)}{9b\gamma - 2\theta^2(1 + \beta)^2}
\] (13)

and the firms expect to under-invest in R&D for all effective spillovers. Substituting (12) into (5), using these in (3) and given \( E(u^2) = \sigma^2 \), expected profits are

\[
E(\pi^C) = E(\pi^C) = \frac{\gamma(\alpha^2 + \sigma^2)}{9b\gamma - 2\theta^2(1 + \beta)^2} > 0
\] (14)

which are positively related to demand volatility, while expected welfare is

\[
E(W^C) = \frac{2\gamma(\alpha^2 + \sigma^2)[18b\gamma - 2\theta^2(1 + \beta)^2]}{9b\gamma - 2\theta^2(1 + \beta)^2}
\] (15)

### 3.1.3 R&D competition v RJV formation (URON v UROC)

Comparing expected R&D levels in (8) and (13),

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\(^{18}\) The positive relationship between expected profits and the volatility of the demand shock is a standard result in the literature. See, among others, Spencer & Brander (1992).

\(^{19}\) There will be some degree of strategic R&D investment given the firms’ output market rivalry. As R&D is a strategic complement at all spillovers, the firms ‘free-ride’ on the R&D of their RJV partner.

\(^{20}\) Each firm’s second-order R&D condition requires \( 9b\gamma - 2\theta^2(1 + \beta)^2 > 0 \).
When $\beta < \frac{1}{2}$ the profit shifting effect of R&D competition dominates the spillover internalisation effect of RJV formation and competitive R&D levels are higher. Conversely, when $\beta > \frac{1}{2}$, the reverse holds and RJV levels are higher.

Comparing expected profit levels in (9) and (14),

$$E(\pi^C) > E(\pi^N) \text{ when } \beta \left(\begin{array}{c} > \\ < \\ > \end{array}\right) \frac{1}{2}$$

so that the firms expect a RJV to be at least as profitable as R&D competition. An interesting finding from (9) and (14) is that changes in the volatility of demand have a greater effect on RJV profits for all effective spillovers, except when $\beta = \frac{1}{2}$ where the effect is identical. When the firms form a RJV, the variance of the demand shock has a positive effect on both own and RJV partner profits. When $\beta = \frac{1}{2}$, R&D is neither a strategic substitute nor complement so that only own profits are affected by demand volatility.

Comparing expected welfare levels in (10) and (15), the second-order conditions ensure that

$$E(W^N) > E(W^C) \text{ when } \beta \left(\begin{array}{c} > \\ < \\ > \end{array}\right) \frac{1}{2}$$

so that a RJV is expected to reduce (increase) welfare relative to the competitive R&D case when $\beta < (>) \frac{1}{2}$. For sufficiently low spillovers, the higher R&D of the competitive R&D case leads to higher output and lower prices so that the higher consumer surplus dominates the higher profits of the RJV case. As spillovers increase, the higher R&D of the RJV case combined with higher profits leads to greater welfare levels.

### 3.2 Pre-uncertainty investment (both ‘jump’) (RUON & RUOC)

This is also a three-stage game, though R&D investment is now chosen to maximise expected profits as it occurs before the demand uncertainty is resolved. As the firms remain output market rivals, ex-post output expressions are again given by (5).\(^{21}\)

#### 3.2.1 R&D competition (RUON)

As the firms are risk-neutral, the representative firm’s first-order R&D condition is

\(^{21}\)In what follows, the subscript ‘u’ denotes that this is a game in which at least one firm undertakes its R&D investment before the demand uncertainty is resolved.
\[
\frac{d[E(\pi)]}{dx} = E\left[\frac{d\pi}{dx}\right] = E\left[\frac{\partial \pi}{\partial x} + \frac{\partial \pi}{\partial q} \cdot \frac{\partial q^*}{\partial x}\right] = 0
\] (19)

Ex-post symmetry implies \( \theta = \theta^* \) and \( \beta = \beta^* \) so that given (1), (2) and (5), (19) can be expressed as

\[
\left[\frac{20(2-\beta)}{3}\right]E(q_u^N) = \mu_u^N E(q_u^N) = \rho_u^N
\] (20)

where \( \mu_u^N \) is the marginal private return to R&D per unit of output. Non-strategic R&D investment requires \( \mu_u^N = 0 \) so the firms expect to over (under) invest in R&D when \( \beta < (>) \frac{1}{2} \).

Given (5), (20) and \( E(u) = 0 \), R&D levels are

\[
x_u^N = x_u^N = \frac{2\alpha \theta(2-\beta)}{9b\gamma - 2\theta^2(2-\beta)(1+\beta)}
\] (21)

which are identical to those in (8), given the assumption of risk-neutrality and \( E(u) = 0 \).

To solve for expected profits levels, use (3), (5), (21) and \( E(u^2) = \sigma^2 \) to derive

\[
E(\pi_u^N) = E(\pi_u^N) = \frac{\gamma\alpha^2[9b\gamma - 2\theta^2(2-\beta)]}{9b\gamma - 2\theta^2(2-\beta)(1+\beta)} + \frac{\sigma^2}{9b} > 0
\] (22)

which is positively related to the volatility of the demand shock, though the effect differs from when both firms wait as R&D levels are not now directly affected by the demand shock. From (4), (5), (21) and (22), expected welfare is

\[
E(W_u^N) = \frac{2\gamma\alpha^2[18b\gamma - 2\theta^2(2-\beta)^2]}{9b\gamma - 2\theta^2(2-\beta)(1+\beta)} + \frac{4\sigma^2}{9b} > 0
\] (23)

which is also positively related to the volatility of the demand shock.

### 3.2.2 RJV formation (R&D co-operation) (RUOC)

The representative firm’s first-order R&D condition is now

\[
\frac{d[E(\pi + \pi^*)]}{dx} = E\left[\frac{d(\pi + \pi^*)}{dx}\right] = E\left[\frac{\partial \pi}{\partial x} + \frac{\partial \pi}{\partial q^*} \cdot \frac{\partial q^*}{\partial x} + \frac{\partial \pi^*}{\partial x} + \frac{\partial \pi^*}{\partial q^*} \cdot \frac{\partial q^*}{\partial x}\right] = 0
\] (24)

---

Ex-post non-strategic R&D levels are \( x_u^N = x_u^N = \frac{\theta(\alpha + u)}{3b\gamma - \theta^2(1+\beta)} \), so that

\[
x_u^N = x_u^N \quad \text{when} \quad \frac{3b\alpha \theta(1-2\beta)}{9b\gamma - 2\theta^2(2-\beta)(1+\beta)} < 1
\]

\[
x_u^N = x_u^N \quad \text{when} \quad \frac{3b\alpha \theta(1-2\beta)}{9b\gamma - 2\theta^2(2-\beta)(1+\beta)} > 1
\]

If \( \beta = \frac{1}{2} \), then, ex-post, the firms over (under) invest in R&D if the demand shock is negative (positive), though, ex-ante, they expect to invest non-strategically, given \( E(u) = 0 \). When \( \beta < \frac{1}{2} \), the firms, ex-post, over-invest in R&D, except if there are relatively large, positive demand shocks. On the other hand, when \( \beta > \frac{1}{2} \), the firms, ex-post, under-invest in R&D, except if there are relatively large, negative demand shocks. When \( \beta < (>) \frac{1}{2} \) and the shock is sufficiently positive (negative), ex-post non-strategic R&D is relatively high (low) and, as each firm’s investment is based on a zero shock, the firms will be under (over) investing in R&D. This contrasts with the certain demand case where the firms always over (under) invest in R&D when \( \beta < (>) \frac{1}{2} \).
Ex-post symmetry again implies $\theta = \theta^*$ and $\beta = \beta^*$ so that, given (1), (2) and (5), (24) can be expressed as

$$
\left[ \frac{2\theta(1+\beta)}{3} \right] E(q_u^C) = \mu_u^C E(q_u^C) = \gamma_u^C
$$

(25)

where $\mu_u^C$ is the marginal RJV return to R&D per unit of output. Non-strategic R&D investment requires $\mu_u^C = \theta(1+\beta)$, so the firms expect to under-invest in R&D for all spillovers.\(^{23}\) From (5) and (25), expected-profit maximising R&D levels are

$$
x_u^C = x_u^{*C} = \frac{2\alpha \theta (1+\beta)}{9b\gamma - 2\theta^2 (1+\beta)^2}
$$

(26)

which are identical to those in (13).\(^{24}\)

From (3), (5) and (26), and given $E(u^2) = \sigma^2$, expected profits are

$$
E(x_u^C) = E(x_u^{*C}) = \frac{\gamma \alpha^2}{9b\gamma - 2\theta^2 (1+\beta)^2} + \frac{\sigma^2}{9b} > 0
$$

(27)

where the effect of greater demand volatility is again positive. From (4), (5), (26) and (27), expected welfare is

$$
E(W_u^C) = \frac{2\gamma \alpha^2 [18b\gamma - 2\theta^2 (1+\beta)^2]}{[9b\gamma - 2\theta^2 (1+\beta)^2]^2} + \frac{4\sigma^2}{9b} > 0
$$

(28)

which, again, is positively related to the volatility of the demand shock.

### 3.2.3 R&D competition v RJV formation

(RUON V RUOC)

Looking first at R&D levels in (21) and (26),

$$
x_u^N \begin{cases} > \gamma_u^C \text{ when } \beta = \frac{1}{2} \end{cases}
$$

(29)

which is identical to when both firms ‘wait’ in (16).\(^{25}\) Comparing expected profit levels in (22) and (27),

\(^{23}\) Given joint profit maximisation, non-strategic R&D investment occurs where each firm invests in R&D to where the marginal RJV return to R&D (equal to the sum of private returns) equals the marginal RJV cost (equal to marginal private R&D cost).

\(^{24}\) *Ex-post non-strategic* R&D levels are $x_u^C = x_u^{*C} = \frac{(\alpha + u)(1+\beta)}{3b\gamma - (1+\beta)^2}$ so that

$$
\begin{cases}
\gamma_u^C > x_u^{*C} \text{ when } u = \frac{-3b\gamma \alpha}{9b\gamma - 2(1+\beta)^2} < 0
\end{cases}
$$

Ex-post, the firms over-invest in R&D for relatively large negative demand shocks, and under-invest for all non-negative and relatively low negative shocks. For relatively large negative demand shocks, ex-post non-strategic R&D is relatively low. The firms’ R&D decisions, however, are based on a zero demand shock so that, ex-post, the firms may over-invest in R&D. Again, this contrasts with the certain demand case where the firms always under-invest in R&D.

\(^{25}\) The intuition for this in given in Section 3.1.3.
so that a RJV is again expected to be at least as profitable as R&D competition.

Interestingly, the effect of the variance of the demand shock on expected profits is identical in both the competitive and co-operative R&D cases. This is due to the fact that the firms’ R&D levels are not directly affected by the true demand shock as they are chosen before the shock is known. Only output levels are directly affected by the shock and as, from (5) and ex-post symmetry, this effect is identical for each firm, demand volatility has an equal effect on expected profits in both cases.\(^{26}\)

Comparing expected welfare levels in (23) and (28), stability conditions imply that

\[
E(W_u^N) = E(W_u^C) \quad \text{when } \beta = \frac{1}{2}
\]

so that, again, a RJV is expected to be welfare reducing (increasing) relative to competitive R&D when \(\beta < (>) \frac{1}{2}\).

4. Sequential R&D: Accommodation

This section looks at where one firm undertakes its R&D investment before the other firm invests in its R&D and the demand uncertainty is resolved. When choosing its R&D, the leader maximises expected own or joint profits subject to the R&D reaction function of the follower by investing at the ‘Stackelberg’ point in R&D space. Ex-ante, the leader expects the

---

\(^{26}\) We can derive ex-post profits and show that

\[
\pi_u^N > < \pi_u^C \quad \text{if } (2\beta - 1)[9\beta\gamma(2\beta - 1) + 2(1 + \beta)(9\beta\gamma - 2(2 - \beta)(1 + \beta)] > 0.
\]

As R&D levels are identical when \(\beta = \frac{1}{2}\), so too are ex-post profits, irrespective of the demand shock. When \(\beta < (>) \frac{1}{2}\), profits in the RJV (competitive R&D) case are higher if the demand shock is non-positive (non-negative), while for positive (negative) shocks, which profit level dominates will depend on the levels of spillovers, the demand shock, the extent of demand relative to exogenous marginal cost (\(\alpha\)), and on the relative ineffectiveness of R&D (\(\beta\)). For example, when \(\beta < \frac{1}{2}\), competitive R&D levels exceed RJV levels. When the shock is negative, the firms over-invest in R&D if they compete in R&D and under (over) invest in R&D when the shock is relatively low (high) if they form a RJV. For relatively large positive shocks, the competitive R&D firms will invest in R&D relatively efficiently, while the co-operative firms will under-invest to a relatively large degree. The effect is to reduce the competitive firms’ marginal production costs to a degree that is not offset by higher R&D costs so profits are increased relative to when the firms form a RJV. On the other hand, a relatively low positive shock implies that RJV profits are higher as the benefit of lower R&D costs dominates any disadvantage in relation to lower marginal production costs. The opposite can be shown when \(\beta > \frac{1}{2}\) and R&D is greater when the firms co-operate in R&D. The interesting point is that compared to the certain demand case, competitive R&D profits may exceed RJV profits, ex-post, over a range of positive (negative) demand shocks when \(\beta < (>) \frac{1}{2}\). This contradicts the D’Aspremont and Jacquemin result that RJV formation is at least as profitable as R&D competition, though this is what the firms expect ex-ante.
follower’s R&D reaction function to be in a position consistent with a zero demand shock. For the follower, however, a non-zero demand shock will, relative to the leader’s expectation, change its best R&D response to any R&D of the leader. For positive shocks, which firm’s R&D level will be greater will depend on the actual value of the demand shock and on spillover levels. In what follows, the leader is denoted by L and the follower by F.

4.1 output stage

The inverse linear demand function and marginal production cost functions are now
\[ p = a - b(q^L + q^F) + u \]
\[ c^L = A - \theta_0(x^L + \beta_1x^F) \geq 0 \quad c^F = A - \theta_1(\beta^L x^L + x^F) \geq 0 \]
while respective R&D costs are
\[ \Gamma^L(x^L) = \gamma(x^L)^2/2 \quad \text{and} \quad \Gamma^F(x^F) = \gamma(x^F)^2/2 \]

As the firms remain output market rivals, the output expressions in (5) are amended to
\[
\begin{bmatrix}
q^L \\
q^F
\end{bmatrix} = \frac{1}{3b} \left[ (\alpha + u) + (2\theta^F - \theta^L \beta^F) x^L + (2\theta^L \beta^F - \theta^F) x^F \right]
\]

The firms’ profits can be expressed as
\[ \pi^i = (p - c^i)q^i - \frac{\gamma(x^i)^2}{2} = \left[ a - bq^i - bq^j - (A - \theta^i x^i - \theta^j \beta^i x^i) \right]q^i - \frac{\gamma(x^i)^2}{2}, \quad (i, j = L, F, i \neq j) \]
while welfare is
\[ W = \frac{bQ^2}{2} + \pi^L + \pi^F \]

4.2 Accommodation: R&D competition (RURON)

The follower’s first-order condition is
\[ \frac{d\pi^F}{dx^F} = \frac{\partial\pi^F}{\partial x^F} + \frac{\partial\pi^F}{\partial q^L} \cdot \frac{\partial q^L}{\partial x^F} = 0 \]
that, from (32) and (34), can be expressed as
\[ \left[ \frac{2(2\theta^F - \theta^L \beta^L)}{3} \right] q_a^{FN} = \mu_u^{FN} q_a^{FN} = \gamma a^{FN} \]
where \( \mu_u^{FN} \) is the follower’s marginal private return to R&D per unit of output. Non-strategic investment requires \( \mu_u^{FN} = \theta^F \) so the follower over (under) invests in R&D when

---

27 As (34) is identical to (5) if the ‘leader’ is the representative firm and the follower is the (*) firm, it could be argued that the introduction of new notation is not required. This would be true if the simultaneous and sequential games were looked at in isolation. Later in this paper, however, it will be easier to compare the simultaneous and sequential games if the notation of each game is different.
\( \theta^t \beta^t < (>) \frac{\theta^f}{2} \), irrespective of the level of the demand shock. As the follower becomes relatively more efficient (\( \theta^f \) increases or \( \theta^t \) decreases), the more likely it is to over-invest in R&D to profit-shift from the leader for any spillover parameter. Using (34) in (38), the follower’s R&D reaction function is

\[
x_{u}^{FN}(q_{u}^{LN}) = \frac{2(2\theta^f - \theta^t \beta^t)(\alpha + u) + (2\theta^f \beta^t - \theta^t)\beta^L \gamma^L}{9b\gamma - 2(2\theta^f - \theta^t \beta^t)^2}
\]  

(39)

so that the leader’s R&D is a strategic substitute (complement) for the follower’s R&D when \((2\theta^f - \theta^t \beta^t)(2\theta^f \beta^t - \theta^t) < (>) 0\), as the firms’ R&D reaction functions are downward (upward) sloping in R&D space.\(^{28}\)

The leader’s first-order R&D condition is

\[
\frac{d [E(\pi^L)]}{dx^L} = E \left[ \frac{\partial \pi^L}{\partial x^L} + \frac{\partial \pi^L}{\partial \gamma^L} \right] = 0
\]

(40)

Ex-post symmetry implies that \( \theta^t = \theta^f \equiv \theta \) and \( \beta^L = \beta^f \equiv \beta \), so that given (32) and (34), (40) can be expressed as

\[
\left\{ \frac{2\theta(2 - \beta)b\gamma - 6\theta^2 (1 - \beta^2)}{3b\gamma - 2\theta^2 (2 - \beta)^2} \right\} E(q_{u}^{LN}) = \mu_{u}^{LN} E(q_{u}^{LN}) = \gamma_{u}^{LN}
\]

(41)

where \( \mu_{u}^{LN} \) is the leader’s marginal return to R&D per unit of output. Non-strategic R&D investment requires \( \mu_{u}^{LN} = \theta \) so the leader expects to over (under) invest in R&D when \( \theta(2\beta - 1)b\gamma - 6\theta^2 \beta(2 - \beta) < (>) 0 \). When \( \beta < \frac{1}{2} \), the leader expects to over-invest in R&D for any unit R&D cost.\(^{29}\) On the other hand, when \( \beta > \frac{1}{2} \), the leader expects to under-invest in R&D, except for a narrow range of relatively high levels of R&D effectiveness where, given the second-order conditions, \( b\gamma < \frac{6\theta^2 \beta (2 - \beta)}{9} \), at which it expects to over-invest in R&D.

When R&D is highly effective (\( b\gamma \) low), the leader over-invests in R&D, even at relatively high effective spillovers, to reduce marginal production costs and shift profits from the follower.

From (34), (40) and (41), the leader’s expected profit maximising R&D level is

\[
x_{u}^{LN} = \frac{2\theta(2 - \beta)b\gamma - 6\theta^2 (1 - \beta^2)}{9b\gamma - 2\theta^2 (2 - \beta)^2}
\]

(42)

so that from (42) and (39), the follower’s profit maximising R&D level is

\[
x_{u}^{FN} = \frac{2\theta(2 - \beta)b\gamma - 6\theta^2 (1 - \beta^2)}{9b\gamma - 2\theta^2 (2 - \beta)^2} \quad \text{and} \quad \frac{2\theta(2 - \beta)}{9b\gamma - 2\theta^2 (2 - \beta)^2}
\]

(43)

which is positively related to the demand shock. Comparing leader and follower R&D levels,

\(^{28}\) The follower’s second-order condition requires \( 9b\gamma - 2(2\theta^f - \theta^t \beta^t)^2 > 0 \).

\(^{29}\) Given the stability conditions, \( 9b\gamma - 6\theta^2 \beta (2 - \beta) > 0 \) when \( \beta < \frac{1}{2} \).
Given ex-post symmetry, then from (39), R&D is neither a strategic substitute nor complement when $\beta = 1/2$, so that in the absence of a shock ($u = 0$), R&D levels are identical as the Cournot and Stackelberg equilibrium points coincide.\(^{30}\) For non-zero shocks, leader R&D will be greater (lower) than follower R&D for negative (positive) shocks, as the follower’s R&D reaction function shifts to the left (right) in R&D space. When $\beta \neq 1/2$, the leader’s R&D exceeds that of the follower for non-positive demand shocks. This is for two reasons. Firstly, the leader’s Stackelberg point implies a greater level of R&D than at the Cournot equilibrium level. Secondly, the leader undertakes its R&D with an expected shock level of zero, while the follower’s R&D is based on the actual shock. For positive shocks, however, which R&D level dominates will depend on the size of the parameters of the model as the follower’s R&D reaction function shifts to the right in $x^L$ space. In ‘jumping’, there is a trade-off between being a Stackelberg R&D leader and being unable to choose the ex-post profit maximising R&D level. If the shock is positive, the follower’s ex-post R&D level may exceed the leader’s ex-ante level given that it waits until the uncertainty is resolved. This is more likely the higher is the actual shock, as the leader’s choice is based on a zero shock.

Using (34) and (42), the leader’s expected profit is

$$E(x^L_u) = \frac{\gamma x^2}{9b} \left[ b_y - 6\theta^2 (2 - \beta)(1 - \beta) \right] + \frac{\sigma^2}{9b} \frac{b_y - 6\theta^2 (2 - \beta)(1 - \beta)}{9b^2 + 2\theta^2 (2 - \beta)^2 / 3}$$

which is again positively related to demand shock volatility. The follower’s expected profit is

$$E(x^F_u) = \frac{\gamma x^2}{9b} \left[ b_y - 6\theta^2 (2 - \beta)(1 - \beta) \right] + \frac{\sigma^2}{9b} \frac{b_y - 6\theta^2 (2 - \beta)(1 - \beta)}{9b^2 + 2\theta^2 (2 - \beta)^2 / 3}$$

with the magnitude of the demand volatility effect different from that of the leader as, for the follower, both output and R&D are directly affected, while only the leader’s output is directly affected by the shock. From (45), (46) and the second-order conditions, increased volatility has a greater effect on the expected profits of the follower.

Comparing (45) and (46),

\(^{30}\)The leader’s second-order R&D condition requires

$$9b_y \left[ b_y - 2\theta^2 (2 - \beta)^2 / 2 \right] + 2\theta^2 (2 - \beta)^2 \left[ b_y - 6\theta^2 (1 - \beta^2) \right] > 0.$$  

\(^{31}\)If $\beta < 1/2$, follower R&D is a strategic substitute (complement) for leader R&D if $b_y > (\leq) \frac{6\theta^2 (1 - \beta^2)}{9}$. Conversely, if $\beta > 1/2$, it is a strategic substitute (complement) when $b_y < (\geq) \frac{6\theta^2 (1 - \beta^2)}{9}$. Given the second order conditions, follower R&D is always a strategic substitute (complement) for leader R&D when $\beta < (\geq) 1/2$. 

17
so that given the second-order conditions, when $\beta < \frac{1}{2}$ and for given unit R&D costs, there is an expected first (second) mover advantage when demand volatility is sufficiently low (high), given that demand volatility has a greater effect on the follower’s expected profits. Conversely, when $\beta > \frac{1}{2}$, there is an expected second-mover advantage for any volatility level. While the leader’s R&D may be expected to be higher, the lower R&D expenditure and relatively large spillover gain of the follower, combined with the larger effect of demand volatility, will offset any advantage in choosing R&D before one’s rival.

Using (42) and (43) in (34), and given (45) and (46), an expression for expected welfare can be derived. As this expression is relatively cumbersome and no simple conclusion can be drawn from it, it is omitted from this section.\textsuperscript{32}

\section*{4.3 Accommodation: RJV formation (R&D co-operation) (RUROC)}

The follower’s first-order R&D condition is

$$
\frac{d(\pi_L^L + \pi_F^F)}{dx^F} = \left[ \frac{\partial \pi_L^L}{\partial x^F} + \frac{\partial \pi_L^L}{\partial q^F} \frac{\partial q^L}{\partial x^F} \right] + \left[ \frac{\partial \pi_F^F}{\partial x^F} + \frac{\partial \pi_F^F}{\partial q^L} \frac{\partial q^L}{\partial x^F} \right] = 0
$$

so that given (32) and (34), we can derive

$$
\left[ \theta^L \beta^L - \frac{2\theta^F - \theta^L \beta^L}{3} \right] q^L_{CL} + \left[ \theta^F - \frac{2\theta^L \beta^L - \theta^F}{3} \right] q^F_{CL} = \mu^L_{CL} q^L_{uL} + \mu^F_{CL} q^F_{uL} = p_{CL}^L
$$

Non-strategic R&D investment requires $\mu^L_{CL} = \mu^F_{CL} = \theta^L \beta^L + \theta^F$ so that, given the leader’s R&D, the follower will under-invest in R&D for all spillovers. Given (34) and (49), the follower’s joint-profit maximising R&D reaction function is

$$
x^L_{uL} (x^L_{CL}) = \frac{2(\theta^L + \theta^F) (\alpha + u) + \frac{1}{2} (2\theta^L \beta^L - \theta^F + \theta^F \beta^L) + (2\theta^L - \theta^F \beta^L - \theta^F \beta^L)}{9b \gamma - 2(2\theta^L \beta^L - \theta^F)^2 - 2(2\theta^F - \theta^F \beta^L)^2} \left( x^L_{uL} \right)
$$

so that the leader’s R&D is a strategic substitute (complement) for that of the entrant’s R&D when $\beta^L < > \frac{\theta^F (4\theta^L - 5\theta^F \beta^L)}{\theta^F (5\theta^L - 4\theta^F \beta^L)} \textsuperscript{33}$

The leader’s first-order R&D condition is now

\begin{multline*}
E(x^L_{CL} < x^L_{CL}) \frac{\partial \pi_L^L}{\partial x^L_{CL}} + \frac{\partial \pi_L^L}{\partial q^L} \frac{\partial q^L}{\partial x^L_{CL}} > 0
\end{multline*}

\textsuperscript{32} In Section 7, restrictions are imposed on the parameters of the model to enable simulation of the model in order to facilitate comparison of the different games.

\textsuperscript{33} The follower’s second-order condition now requires $9b \gamma - 2(2\theta^L \beta^L - \theta^F)^2 - 2(2\theta^F - \theta^F \beta^L)^2 > 0$.
\[
dE(\pi^L + \pi^F) = E \frac{d\pi^L}{dx^L} + E \frac{d\pi^F}{dx^L} = E \frac{\partial \pi^L}{\partial x^L} + \frac{\partial \pi^F}{\partial x^L} + E \frac{\partial \pi^L}{\partial q^L} \cdot \frac{\partial q^L}{\partial x^L} + \frac{\partial \pi^F}{\partial q^L} \cdot \frac{\partial q^L}{\partial x^L} = 0 \quad (51)
\]

where the first term on the right-hand side of (51) is given by (40). Given (48), the expression in (51) can be reduced to

\[
dE(\pi^L + \pi^F) = E \frac{\partial \pi^L}{\partial x^L} = 0 \quad (52)
\]
as the R&D reaction function effects are eliminated given the shape of the iso-profit curves when the firms form a RJV (see Figure 1). Ex-post symmetry implies that \(\theta^L = \theta^F = \theta\) and \(\beta^L = \beta^F = \beta\), so that given (32), (33) and (34), (52) can be expressed as

\[
\left[\frac{2\theta(2-\beta)}{3}\right]E(q_{u}^{LC}) + \left[\frac{2\theta(2-1)}{3}\right]E(q_{u}^{FC}) = \mu_{u}^{L}E(q_{u}^{LC}) + \mu_{u}^{F}E(q_{u}^{FC}) = 0 \quad (53)
\]

Non-strategic R&D investment requires \(\mu_{u}^{LC} = \mu_{u}^{F} = \theta(1+\beta)\) so that the leader will expect to under-invest in R&D for any spillover level. Given (34) and (39), the leader’s expected joint profit maximising R&D level\(^{35}\) is

\[
x_{u}^{LC} = \frac{2\alpha\theta(1+\beta)}{9b\gamma - 2\theta^2(1+\beta)^2} \quad (54)
\]

Substituting (54) into (50), the follower’s profit maximising R&D level is

\[
x_{u}^{FC} = \frac{2\alpha\theta(1+\beta)}{9b\gamma - 2\theta^2(1+\beta)^2} + \frac{2\theta(1+\beta)}{9b\gamma - 2\theta^2(2-\beta)^2 - 2\theta^2(2\beta-1)^2} u \quad (55)
\]

which, again, is positively related to the demand shock. From (54) and (55), the follower’s R&D will be greater (lower) than that of the leader when the demand shock is positive (negative), though the leader expects them to be identical when undertaking its R&D investment. Using (54) and (55) in (34) and (35), the leader’s expected profit is

\[
E(\pi_{u}^{LC}) = \frac{\gamma\alpha^2}{9b\gamma - 2\theta^2(1+\beta)^2} + \frac{\sigma^2}{9b} \left[\frac{\theta b\gamma - 6\theta^2(2-\beta)(1-\beta)}{9b\gamma - 2\theta^2(2-\beta)^2 - 2\theta^2(2\beta-1)^2}\right] > 0 \quad (56)
\]

which is positively related to the volatility of demand. The follower’s expected profit is

\[
E(\pi_{u}^{FC}) = \frac{\gamma\alpha^2}{9b\gamma - 2\theta^2(1+\beta)^2} + \frac{\sigma^2}{9b} \left[\frac{\theta b\gamma + 6\theta^2(2\beta-1)(1-\beta)}{9b\gamma - 2\theta^2(2-\beta)^2 - 2\theta^2(2\beta-1)^2}\right] - 18\beta\gamma\theta^2(1+\beta)^2 \quad (57)
\]

where demand volatility again has a different effect on the follower’s profits given that the demand shock does not directly affect the leader’s R&D. Comparing (56) and (57),

\[
E(\pi_{u}^{LC}) > E(\pi_{u}^{FC}) \text{ if } b\gamma(5-7\beta) < 6\theta^3(1-\beta)^3 \quad (58)
\]

\(^{34}\) The shape of the iso-profit curves ensures that the Stackelberg equilibrium in the sequential R&D game is identical to the Cournot-Nash equilibrium in the simultaneous R&D game.

\(^{35}\) The leader’s second-order R&D condition requires \([9b\gamma - 2\theta^2(1+\beta)^2][9b\gamma - 18\theta^2(1-\beta)^2] > 0\).
The relationship between expected profit levels depends only on the level of spillovers and the relative ineffectiveness of R&D. Given the second-order conditions, then when $\beta < 5/7$, there will be an expected second-mover advantage, except for a narrow range of spillovers at which there is a first-mover advantage (see Figure 2). On the other hand, when $\beta > 5/7$, there will be an expected first-mover advantage for all levels of R&D effectiveness.

Using (54) and (55) to derive the firms’ output levels in (34), and given (56) and (57), an expression for expected welfare can be derived. Again, this expression is relatively large and as no simple conclusion can be drawn, it is omitted from the analysis.

### 4.4 Accommodation: R&D competition vs RJV formation (RURON vs RUROC)

Looking firstly at R&D levels in (42) and (54),

$$x^L_u > x^L_C \text{ if } (2\beta-1) \left[ 8b \gamma^2 - 36b \gamma^2 (2-\beta)^2 + 4\theta^4 (2-\beta)(1+\beta)(7\beta^2 - 13\beta + 7) \right] < 0 \quad (59)$$

Given the stability conditions, RJV R&D is greater than (equal to) competitive R&D when $\beta > (=) \frac{1}{2}$, despite the possible over-investment when competing in R&D. For low spillovers ($\beta < \frac{1}{2}$), absence of a shock as R&D levels are identical. For non-zero shocks, however, and given the second order conditions, the leader’s (follower’s) profits are higher if the demand shock is negative (positive) as the leader’s R&D is based on a shock of zero while the follower’s R&D reaction function moves to the left (right) in R&D space and its joint profit maximising R&D level is lower (higher) than the leader’s. When $\beta < \frac{1}{2}$ then, the leader’s profits are higher for relatively large negative shocks, while the follower’s are higher for all other shocks. For large negative shocks, the leader’s R&D is higher than the follower’s as the follower’s R&D reaction function shifts to the left in R&D space. For the leader, the benefit of lower marginal production costs offsets higher R&D costs so that profits are higher.

$$x^L_u > x^L_C \text{ if } a = \frac{4\theta \gamma b (2\beta-1) \gamma b (2-\beta)^2 - 2\theta^2 (2\beta - 1)^2}{\gamma b (5-7\beta) - 6\theta (1-\beta)}.$$  

When $\beta = \frac{1}{2}$, ex-post profits are identical in the absence of a shock as R&D levels are identical. For non-zero shocks, however, and given the second order conditions, the leader’s (follower’s) profits are higher if the demand shock is negative (positive) as the leader’s R&D is based on a shock of zero while the follower’s R&D reaction function moves to the left (right) in R&D space and its joint profit maximising R&D level is lower (higher) than the leader’s. When $\beta < \frac{1}{2}$ then, the leader’s profits are higher for relatively large negative shocks, while the follower’s are higher for all other shocks. For large negative shocks, the leader’s R&D is higher than the follower’s as the follower’s R&D reaction function shifts to the left in R&D space. For the leader, the benefit of lower marginal production costs offsets higher R&D costs so that profits are higher.

### Footnotes

36 Ex-post profits are

$$\pi^{LC}_u = \frac{\gamma a}{9\theta b^2 - 2\theta^2 (1+\beta)^2} \left[ \gamma a + \frac{8b \gamma b (2\beta-1) \gamma b (2-\beta)^2 - 2\theta^2 (2\beta - 1)^2}{\gamma b (5-7\beta) - 6\theta (1-\beta)} \right]$$

and

$$\pi^{FC}_u = \frac{\gamma (a-A)}{9\theta b^2 - 2\theta^2 (1+\beta)^2} \left[ (a-A) + \frac{8b \gamma b (2\beta-1) \gamma b (2-\beta)^2 - 2\theta^2 (2\beta - 1)^2}{\gamma b (5-7\beta) - 6\theta (1-\beta)} \right].$$ 

When $\beta = \frac{1}{2}$, ex-post profits are identical in the absence of a shock as R&D levels are identical. For non-zero shocks, however, and given the second order conditions, the leader’s (follower’s) profits are higher if the demand shock is negative (positive) as the leader’s R&D is based on a shock of zero while the follower’s R&D reaction function moves to the left (right) in R&D space and its joint profit maximising R&D level is lower (higher) than the leader’s. When $\beta < \frac{1}{2}$ then, the leader’s profits are higher for relatively large negative shocks, while the follower’s are higher for all other shocks. For large negative shocks, the leader’s R&D is higher than the follower’s as the follower’s R&D reaction function shifts to the left in R&D space. For the leader, the benefit of lower marginal production costs offsets higher R&D costs so that profits are higher.

37 There seems to be little economic intuition as to why a spillover level of $5/7$ is a threshold that induces a leader’s expected profits to exceed those of a follower.

38 This expression will be simulated in Section 7 by imposing restrictions on the model parameters.
½), however, the [...] term in (59) is positive when \( \beta = 0 \) and increasing in the spillover so that competitive R&D exceeds RJV levels at these spillovers.

For the follower,

\[
x_{u}^{FN} < x_{u}^{EC} \quad \text{if} \quad \beta > -\frac{9\theta y_{u}^{2\theta}(2-\beta)^{2}(4-5\beta)}{6\theta y_{u}^{2\theta}(1+\beta)^{2}+y_{u}^{2\theta}(2-\beta)(1-\beta)^{2}+2y_{u}^{2\theta}(1+\beta)^{2}-2y_{u}^{2\theta}(2-\beta)^{2}(1+\beta)^{2}}
\]

(60)

For all spillovers, follower R&D under RJV formation tends to exceed that of R&D competition for all non-negative shocks and relatively low negative shocks, given likely over-investment by a leader when competing in R&D. For relatively high negative demand shocks, follower R&D under RJV formation may be so low that its competitive level exceeds it.

The more interesting comparison is between a leader’s expected profits, as this indicates the likelihood of the firms forming a RJV. From (45) and (56),

\[
E(x_{u}^{LN}) < E(x_{u}^{EC}) \quad \text{if} \quad \beta > -\frac{b_{y}y_{u}^{2\theta}(2-\beta)(4-5\beta)}{b_{y}y_{u}^{2\theta}(1+\beta)^{2}+y_{u}^{2\theta}(2-\beta)(1-\beta)^{2}+2y_{u}^{2\theta}(1+\beta)^{2}-2y_{u}^{2\theta}(2-\beta)^{2}(1+\beta)^{2}}
\]

(61)

Given the second–order conditions, the expression on the right hand side of (61) is negative for all spillovers, so that for any level of demand volatility, a leader will expect a RJV to be more profitable than R&D competition when R&D levels are chosen sequentially.\(^{39}\)

5. Accommodation: Simultaneous v sequential R&D investment

5.1 R&D investment: R&D Competition

Comparing expected R&D levels in (8), (21) and (42),

\[
E(x_{u}^{N}) = E(x_{u}^{LN}) < E(x_{u}^{EC}) \quad \text{when} \quad \beta > \frac{z}{2}
\]

(62)

so that a R&D leader expects to undertake at least as high a R&D level relative to when the firms simultaneously choose their R&D. As the leader expects to have at least as high a R&D level as the follower, then

\[
E(x_{u}^{N}) = E(x_{u}^{FN}) \quad \text{when} \quad \beta = \frac{z}{2}
\]

(63)

---

\(^{39}\) Given the second-order conditions, it cannot be the case that \( b_{y} < \frac{2\theta y^{2}(2-\beta)(4-5\beta)}{9} \) and \( b_{y} < \frac{2\theta y^{2}(1+\beta)^{2}}{9} \) for any spillovers.
so a follower’s R&D level is also expected to exceed simultaneous R&D levels when \( \beta > \frac{1}{2} \).

Comparing actual R&D levels in (7) and (21), R&D levels when both firms ‘wait’ are greater (lower) than when both ‘jump’ if the demand shock is positive (negative). The interesting question is how a leader’s R&D compares to where both firms ‘wait’.\(^\text{40}\) From (42) and (7),

\[
x^u_{LN} \begin{cases} 
> x^N \text{ when } \alpha > 0 \\
< x^N \text{ when } \alpha < 0
\end{cases}
\]

When \( \beta = \frac{1}{2} \), R&D when both firms ‘wait’ is greater (lower) than a leader’s when the demand shock is positive (negative), with identical R&D levels if there is no shock. For all other spillovers, leader R&D will be higher for all non-positive shocks. For positive shocks, however, leader R&D will be greater for relatively low shocks, while R&D when ‘waiting’ will be higher for relatively large shocks. This is because the leader’s R&D choice is based on an expected shock of zero, while the R&D of ‘waiting’ firms is based on the actual shock.

### 5.2 R&D Investment: RJV formation (R&D co-operation)

Comparing expected R&D levels in (13), (26), (54) and (55),

\[
E(x^C) = E(x^C_u) = E(x^LC) = E(x^FC)
\]

which are identical as the Cournot-Nash and Stackelberg points coincide due to the shape of the iso-profit contours under RJV formation. Ex-post R&D investments will be directly affected by the shock which shifts the follower’s R&D reaction function, but not the leader’s. Comparing actual R&D levels in (26), (54), (55) and (12), ex-post R&D is greater (lower) than ex-ante levels if the actual shock is positive (negative), with R&D levels identical if there is no shock as ex-ante R&D levels are based on an expected shock of zero, whereas ex-post R&D is positively related to the shock. The interesting comparison, therefore, is between a follower and when both firms ‘wait’. From (55) and (12),

\[
x^C \begin{cases} 
> x^FC \text{ if } 4 \theta^2 (2 - \beta)(2 \beta - 1) \alpha = 0 \\
< x^FC \text{ otherwise}
\end{cases}
\]

R&D levels when following or when both ‘wait’ are identical for any demand shock when \( \beta = \frac{1}{2} \), as R&D is neither a strategic substitute nor complement at this spillover. When \( \beta < \frac{1}{2} \), R&D levels when ‘waiting’ are higher (lower) than those of a follower when the demand shock is negative (positive). On the other hand, when \( \beta > \frac{1}{2} \), ‘waiting’ leads to a higher

\(^{40}\) When firms choose their R&D before the shock is revealed, expected levels are equivalent to actual levels. Given this and (62), \( x^u_{LN} \begin{cases} 
> x^u \text{ when } \alpha \geq \frac{1}{2} \\
< x^u \text{ when } \alpha < \frac{1}{2}
\end{cases}\).
(lower) R&D level for positive (negative) demand shocks. To illustrate this, suppose that \( \beta < \frac{1}{2} \) and there is a negative demand shock \( (u < 0) \). When both firms ‘wait’, the firms’ downward-sloping R&D reaction functions shift to the left in R&D space and R&D levels decrease by identical amounts. When R&D levels are chosen sequentially, however, only the follower’s R&D reaction function shifts to the left, as the leader’s R&D is consistent with a follower’s R&D reaction function based on an expected shock of zero. This leads to a relatively large fall in the follower’s R&D, as any reduction in total R&D in response to a negative shock is undertaken solely by the follower.

5.3 Expected profits: R&D competition

In comparing expected profits, multiple Nash equilibria are possible in relation to the timing of R&D investment. As both firms have two possible actions, jump or wait, the firms’ payoff matrix is as follows:

<table>
<thead>
<tr>
<th></th>
<th>Jump</th>
<th>Wait</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jump</td>
<td>( E(\pi_u^N) ), ( E(\pi_u^N) )</td>
<td>( E(\pi_u^{LN}) ), ( E(\pi_u^{LN}) )</td>
</tr>
<tr>
<td>Wait</td>
<td>( E(\pi_u^{FN}) ), ( E(\pi_u^{LN}) )</td>
<td>( E(\pi^N) ), ( E(\pi^N) )</td>
</tr>
</tbody>
</table>

Comparing where both ‘jump’ to being a follower in (22) and (46), \( E(\pi_u^N) = E(\pi_u^{FN}) \) if

\[
\sigma > \frac{9b^2y^2(2(2-\beta))}{2b^2(2-\beta)^2(b\theta - 2b^2(2-\beta)(1 + \beta) - \theta(2(2-\beta)(1-\beta))} \tag*{(67)}
\]

When \( \beta = \frac{1}{2} \) and demand volatility is positive, a follower’s expected profit exceeds that of when ‘both jump’. On the other hand, when \( \beta \neq \frac{1}{2} \), which expected profit level dominates will depend on the values of the various parameters of the model. Similarly, a comparison of a leader against ‘both wait’ in (45) and (9), shows that \( E(\pi_u^{LN}) = E(\pi^N) \) if

\[
\sigma > \frac{64b^2y^2(2(2-\beta))}{2b^2(2-\beta)^2(b\theta - 2b^2(2-\beta)(1 + \beta) - \theta(2(2-\beta)(1-\beta))} \tag*{(68)}
\]
where, if $\beta = \frac{1}{2}$, expected profits when both ‘wait’ exceeds those of a leader when demand volatility is positive. For other spillover levels, which expected profit level dominates again depends on the value of model parameters.

Due to the complexity of the expressions in (67) and (68), no simple conclusions can be drawn on the Nash equilibrium outcome. Given this, the expressions must be simulated by imposing restrictions on the parameters of the model. This is left until Section 7.

5.4 Expected Profit: RJV formation (R&D co-operation)

The payoff matrix of the firms is now

\[
\begin{array}{c|cc}
& \text{Jump} & \text{Wait} \\
\hline
\text{Jump} & E(\pi_u^{K}) , E(\pi_u^{Kc}) & E(\pi_u^{Lc}) , E(\pi_u^{Fc}) \\
\text{Wait} & E(\pi_u^{Fc}) , E(\pi_u^{Lc}) & E(\pi_u^{C}) , E(\pi_u^{C}) \\
\end{array}
\]

Comparing where both ‘jump’ to being a follower in (27) and (57),

\[
E(\pi_u^{C}) > E(\pi_u^{Fc}) \text{ if } \lambda = 27b\gamma(1-\beta) - 2\theta^2(2-\beta)[11\beta^2 - 17\beta + 8] \leq 0 \tag{69}
\]

while, from (56) and (14), comparing a leader to when ‘both wait’ implies that

\[
E(\pi_u^{LC}) > \frac{\lambda}{\theta} \text{ if } \rho = 27b^2\gamma^2(1-\beta) + 2b\gamma\theta^2[10\beta^3 - 30\beta^2 + 45\beta - 23] + 4\theta^4(2-\beta)^2(1-\beta)(1-\beta^2) \leq 0 \tag{70}
\]

Again, analysis of the expressions in (69) and (70), to determine the Nash equilibrium, requires imposing restrictions on the parameters of the model. This is left until Section 7.

6. Sequential R&D: Activity prevention

If the leader wishes to prevent activity by the follower, it is assumed that there are two possible scenarios. The leader can choose its R&D investment so that it expects the follower not to produce, or it can ensure that the follower will never produce.\(^{41}\) In the former case, the leader invests in R&D so that the follower’s expected output level is zero. In the latter case, however, the leader undertakes its R&D investment to ensure that the follower’s output level is always zero, irrespective of the size of the demand shock, by assuming that the demand shock will be as large as possible.

\(^{41}\) Ideally, this section would determine optimal activity-preventing behaviour for all possible levels of demand shock. As this is not the main focus of the paper, however, and given that the expected and highest value of the shock are known, only these two cases are analysed.
R&D investment in the expected activity prevention case implies that the leader will only be a monopolist in the output market if the demand shock is non-positive. On the other hand, certain activity prevention implies that the leader will be a monopolist for any demand shock. Ex-post, however, the latter strategy will be sub-optimal for all shocks other than the largest possible shock, as the leader’s activity-preventing R&D level, and the consequent R&D costs, will be ‘too high’. The benefit of always being a monopolist, in terms of higher output market profits, must be compared to the higher R&D costs required to prevent activity for certain, especially if the actual shock is less than its maximum possible level.

It is assumed that the firms always compete in R&D when the leader attempts to prevent activity by the follower as RJV formation is an unrealistic strategy for the leader.

6.1 Output stage and follower R&D

If follower activity is unprofitable, the leader is a monopolist and its output is

$$q^L = \frac{(\alpha + u) + \theta^L x^L}{2b}$$  \hspace{1cm} (71)

while if activity is profitable, output levels are given by (34). The follower’s R&D investment incentive vis-à-vis the leader is unchanged from the entry accommodation case. Consequently, its R&D reaction function is given by (39).

6.2 Leader R&D: expected activity prevention

Substituting (39) into the follower’s output expression in (34), the leader chooses its R&D to ensure that $E(q^F) = 0$. Given firm symmetry, which implies that $\theta^L = \theta^F = \theta$ and $\beta^L = \beta^F = \beta$, and $E(u) = 0$, the leader’s R&D level is

$$x_{E(u)=0}^{LD} = \frac{\alpha}{\theta(1-2\beta)}$$  \hspace{1cm} (72)

which is undefined for $\beta \geq \frac{1}{2}$ as R&D is no longer a strategic substitute at these spillovers and activity prevention is not a viable strategy. The leader’s R&D is not a function of unit R&D costs as a fixed level of R&D must be undertaken to ensure expected activity prevention, irrespective of common unit R&D costs. As output levels in (34) are not directly affected by unit R&D costs, if the follower does not become active its R&D will be zero and so the leader’s output will also not be a function of unit R&D costs. From (72), the leader’s R&D is increasing in the effective spillover to counter-act an increasing gain to the follower for any level of R&D and ensure that activity is expected to be prevented. Substituting (72) into (39), the follower’s profit maximising R&D level is
\[ x_{E(u)=0}^{FD} = \begin{cases} \frac{2\theta(2-\beta)u}{9b\gamma - 2\theta^2(2-\beta)^2} & \text{if } u > 0 \\ 0 & \text{if } u \leq 0 \end{cases} \]  
(73)

Substituting (72) and (73) into (34), the follower’s ex-post profit maximising output level is

\[ q_{E(u)=0}^{FD} = \begin{cases} \frac{3\mu}{9b\gamma - 2\theta^2(2-\beta)^2} & \text{if } u > 0 \\ 0 & \text{if } u \leq 0 \end{cases} \]  
(74)

so that the leader will be a monopolist when the demand shock is non-positive, while the industry will be a duopoly if the shock is positive.

Substituting (72) into (71) and substituting (72) and (73) into (34), we can derive the leader’s expected output. As the demand shock is uniformly distributed, its probability density function is \( f(u) = \frac{1}{2\sigma} \) and the leader’s expected profit maximising output is

\[
E(q_{E(u)=0}^{LD}) = \left\{
\int_{-\infty}^{b} \alpha(1-\beta) + 9b\gamma - 6\theta^2(2-\beta)^2 \frac{du}{4b^2b\gamma - 2\theta^2(2-\beta)^2}
\right\}
(75)

which is negatively related to the maximum demand shock. As the follower only produces output for positive demand shocks, the higher the maximum shock, the higher its expected output and the lower the expected output of the leader, given homogenous goods.

Using \( \pi^2 = 3\sigma^2 \) (see Section 2), the leader’s expected profit is

\[
E(\pi_{E(u)=0}^{FD}) = \left\{\int_{-\infty}^{b} \pi_{E(u)=0}^{FD}(u) f(u) du \right\}
\]

\[
= \frac{\alpha^2(2\beta^2(1-\beta)^2 + b\gamma)}{2b^2\theta^2(1-2\beta)^2} \cdot \frac{\alpha(1-\beta)[9b\gamma - 6\theta^2(2-\beta)^2]}{12b(1-2\beta)[9b\gamma - 2\theta^2(2-\beta)^2]} + \frac{\sigma^2}{72b} + \frac{9b\gamma - 2\theta^2(2-\beta)^2}{[9b\gamma - 2\theta^2(2-\beta)^2]^2} + \frac{4b\gamma - 6\theta^2(2-\beta)(1-\beta)}{[9b\gamma - 2\theta^2(2-\beta)^2]^2}
\]

while, from (73) and (74), the follower’s is

\[
E(\pi_{E(u)=0}^{FD}) = \left\{\int_{0}^{\infty} \pi_{E(u)=0}^{FD}(u) f(u) du \right\}
\]

\[
= \frac{2\sigma^2}{2b\gamma - 2\theta^2(2-\beta)^2}
\]

(66a)

Using (74) and the leader’s output levels in (34) and (71) for non-positive and positive shocks, respectively, to derive the square of total industry output and using this, (76) and (76a), expected welfare is

\[
E(W_{E(u)=0}^{FD}) = \frac{2\alpha^2(3\beta^2(1-\beta)^2 + b\gamma)}{4b^2\theta^2(1-2\beta)^2} + \frac{\alpha(1-\beta)[9b\gamma - 18\theta^2(2-\beta)^2]}{24b(1-2\beta)[9b\gamma - 2\theta^2(2-\beta)^2]} + \frac{\sigma^2}{27b\gamma - 2\theta^2(2-\beta)^2} + \frac{4b\gamma - 6\theta^2(2-\beta)(1-\beta)}{[9b\gamma - 2\theta^2(2-\beta)^2]^2}
\]

(77)

6.3 Leader R&D: certain entry deterrence
The leader now chooses its R&D to ensure that the follower never becomes active for any level of the demand shock by assuming that the demand shock will be at its highest level. The leader, therefore, is always an output market monopolist with its output given by (71).

Substituting (39) into the follower’s output expression in (34), the leader chooses its R&D to ensure that the follower’s ex-post output level \( q^F \) is zero. Again given firm symmetry, this implies that

\[
x_{LD}^{u-a} = \frac{\alpha + \bar{u}}{\theta(1-2\beta)}
\]

(78)

which is again undefined for \( \beta \geq \frac{1}{2} \) as activity prevention is not viable when the leader’s R&D is no longer a strategic substitute for that of the follower. Also, leader R&D is increasing the maximum shock level. Substituting (78) into (39), the follower’s profit maximising R&D is

\[
x_{FD}^{u-a} = \frac{2\theta(2-\beta)(u-\bar{u})}{9b\gamma - 2\theta^2(2-\beta)^2} \leq 0
\]

(79)

so the follower never undertakes any R&D investment as \( u \leq \bar{u} \).\(^{42}\) Given this, substituting (78) into (71) implies that the leader’s monopoly output level is

\[
q_{LD}^{u-a} = \frac{2\alpha(1-\beta) + [\bar{u} + u(1-2\beta)]}{2b(1-2\beta)}
\]

(80)

which is increasing in the maximum level of the demand shock. The higher is the maximum shock, the greater the level of R&D required to ensure that the follower will never become active. As marginal production costs decrease due to higher R&D levels, the profit maximising output of the leader increases. Given (78) and (80), the leader’s expected profit is

\[
E(x_{u-a}^{LD}) = \int_{\bar{u}}^{u} x_{u-a}^{LD}(u) f(u) du = \int_{\bar{u}}^{u} \left\{ b \left[ \frac{2\alpha(1-\beta) + u(1-2\beta) + \bar{u}}{2b(1-2\beta)} \right]^2 \right\} - \frac{1}{2} \left[ \frac{\alpha + \bar{u}}{\theta(1-2\beta)} \right]^2 f(u) du
\]

(81)

As \( f(u) = \frac{1}{2\bar{u}} \) and \( \bar{u}^2 = 3\sigma^2 \), the leader’s expected profit can be expressed as

\[
E(x_{u-a}^{LD}) = \frac{2\alpha^2 [3\theta^2(1-\beta)^2 + b\gamma]}{4b\theta^2(1-2\beta)^2} + 4\alpha [3\theta^2(1-\beta) - b\gamma] 3\sigma^2 + \sigma^2 [3\theta^2 - 2b\gamma + \theta^2(1-2\beta)^2]
\]

(82)

Using (82), \( \bar{u}^2 = 3\sigma^2 \) and the expected value of the square of (80), expected welfare is

\[
E(W_{u-a}^{D}) = \frac{4\alpha^2 [3\theta^2(1-\beta)^2 - b\gamma]}{8b\theta^2(1-2\beta)^2} + 4\alpha [3\theta^2(1-\beta) - b\gamma] 3\sigma^2 + \left[ 3\theta^2 - 2b\gamma + \theta^2(1-2\beta)^2 \right] \sigma^2
\]

(83)

6.4 Activity prevention: certain v expected

From (72) and (78), the leader’s R&D is always higher when it attempts to prevent activity for certain as this R&D level is based on the highest possible shock. While certain

\(^{42}\) The follower’s second-order condition requires \( 9b\gamma - 2\theta^2(2-\beta)^2 > 0 \).
activity prevention ensures that the leader is always a monopolist in the output market, whether this is more profitable than expected activity prevention will depend on whether the expected benefits of being a monopolist for certain offset the higher R&D costs of ensuring that the follower will never become active. From (76) and (82), it can be shown that

\[ E(\pi^{J,J}) \geq E(\pi^{W,W}) \text{ if } \sigma > \frac{2(\gamma - 1) - \beta}{\theta (1 - \beta)} \]

where the denominator in (84) is positive given the second-order conditions. Given the complexity of the expression in (84), a comparison of expected profits in the activity prevention cases is left until Section 7 when restrictions are imposed on the various parameters of the model in order to facilitate simulation. Comparing expected welfare levels in (77) and (83) also leads to a complex outcome and comparison is again left until Section 7.

7. Results

To compare the various cases, it is necessary to simulate the model by imposing restrictions on the parameters of the behavioural functions. For simplicity, b, \( \theta \) and \( \alpha \) are normalised to unity, while \( \beta \), \( \gamma \) and \( \sigma^2 \) are exogenous parameters.\(^{43}\) As \( \alpha - \bar{\alpha} > 0 \) is required to ensure positive output, then \( \bar{\alpha} < 1 \), which puts a known and finite support on the upper bound of the probability distribution of the demand shock. As \( \sigma^2 = \frac{\bar{\alpha}}{3} \), it must be the case that \( \sigma^2 < \frac{1}{3} \). This section mostly looks at where \( \gamma = 2 \) to allows comparison of R&D, profit and welfare levels for most spillover levels.\(^{44}\) Where possible, it also looks at where \( \gamma = 1 \).

7.1 R&D competition: Accommodation

Looking at Figure 3, three Nash equilibria are possible when \( \beta < \frac{1}{2} \).\(^{45}\) For low spillover and volatility levels, both firms ‘jumping’ is a Nash equilibrium as investing before the demand shock is a dominant strategy for both firms. At such low spillovers, a firm that ‘jumps’ expects to over-invest in R&D and shift profits from its rival through lower marginal production costs. For slightly higher, but still relatively low, levels of demand volatility, two Nash equilibria are possible where both firms either ‘jump’ or ‘wait’. Comparing expected

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\(^{43}\) As \( b = \theta = 1 \), the relative effectiveness of R&D is equivalent to the inverse of unit R&D costs. Also, given \( \theta = 1 \), effective spillovers are equivalent to \( \beta \).

\(^{44}\) Results are similar for other admissible values of unit R&D cost.

\(^{45}\) In Figure 3, ‘J’ denotes ‘jump’ while ‘W’ denotes ‘wait’. Given this, (J,J) and (W,W) refer to the Nash equilibria where both firms ‘jump’ and ‘wait’, respectively. On the other hand, (J,W) and (W,J) refer to where the firms sequentially choose R&D levels.
profits when $\gamma = 2$ (see (9) and (22)), and given the second-order conditions, expected profits are higher when both firms wait, given the effect of demand volatility on expected profits. For sufficiently high levels of volatility, both firms ‘waiting’ is a unique Nash equilibrium.

When $\beta = 0.5$, multiple Nash equilibria occur if there is no demand uncertainty ($\sigma^2 = 0$), underlining one of the findings of the previous chapter, as R&D is neither a strategic substitute nor strategic complement at this spillover and there is no first-mover advantage if the firms compete in R&D or form a RJV. For any positive level of demand uncertainty, however, both firms waiting is again a Nash equilibrium.

When spillovers are relatively high ($\beta > 0.5$), three Nash equilibria are possible. For very low levels of demand volatility, a sequential R&D Nash equilibrium is possible, with the range of demand volatility at which this occurs increasing in the spillover. By waiting, each firm can benefit from its rival’s R&D through relatively high spillovers, without incurring the higher R&D costs. This incentive increases in the spillover. For all other levels of demand volatility, the Nash equilibrium is where both firms ‘wait’. If one firm waits, the other also waits as being a leader increases R&D costs to an extent that is not offset by an increase in revenue as the leader’s marginal production costs remain relatively close to the follower’s.

### 7.2 R&D Competition: Activity prevention

Using (84), it can be seen from Figure 4 that as $\sigma^2 < 1/3$, so that $-0.577 < \sigma < 0.577$, expected profits from expecting to prevent activity exceed those of doing so for certain. As spillovers increase, the demand volatility threshold at which certain prevention is expected to become more profitable is increasing due to the higher R&D required to prevent activity for certain. When $\gamma = 1$, certain prevention is expected to be more profitable for a small range of spillover and demand volatility levels (see Figure 5). Lower unit R&D costs imply that the increased R&D expenditure required to prevent activity for certain is more than offset by the increased output market profit and expected profits are higher.

In general, for any spillover, the lower are unit R&D costs, the lower is the threshold level of demand volatility at which certain prevention is expected to become more profitable.

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46 This comparison can only be made for $\beta < 0.5$ as activity prevention is not viable for $\beta \geq 0.5$.

47 As unit R&D costs increase, the result is generally identical, though the threshold level of demand volatility required to ensure that certain deterrence is expected to be more profitable is increasing.

48 It is possible to look at the activity prevention case for $\gamma = 1$ as only the follower’s second-order R&D condition is required to be satisfied. The leader does not need to satisfy its own second-order R&D condition when it chooses its R&D to prevent follower activity.
Similarly, for any unit R&D cost, as the spillover increases, the increasing gain to the follower requires increased R&D expenditure in order to ensure certain prevention and this offsets the benefit of being an output market monopolist.

While the above analysis determines which activity-preventing strategy is preferable to a leader, such actions will only be undertaken if expected profits are positive. In Figure 6 where $\sigma^2 = 0.05$, expected profits from expected activity prevention (see (76)) are negative for all spillovers for any $\gamma \geq 2$. Consequently, given the analysis of Figure 4, expected profits from certain prevention are also negative at these unit R&D cost levels so that neither activity-preventing strategy will be undertaken. On the other hand, when $\gamma = 1$, expected activity prevention is expected to be profitable for $\beta \leq 0.2$, while from Figure 7, certain activity prevention (see (82)) is also expected to be profitable when $\beta \leq 0.2$, but non-profitable for all spillovers when $\gamma \geq 2$. Comparing expected profit levels in Figure 8, certain prevention is expected to be more profitable when $\beta \leq 0.1$, as the benefits of becoming a monopolist, in terms of higher output market profits and lower required R&D investment, dominate, with expected prevention dominating for all other relevant spillovers.

Activity prevention, be it expected or certain, is only profitable when the leader’s R&D is highly effective ($\gamma$ is low) and spillovers are relatively low. As spillovers increase, then for a given demand variance and unit R&D cost, so must the leader’s R&D investment to ensure that the follower does not become active. As unit R&D costs increase, both activity prevention strategies become less profitable, while for given levels of unit R&D cost and spillovers, each strategy becomes less profitable as the volatility of demand increases. Higher volatility is equivalent to a higher maximum shock so that greater R&D investment is required in both activity prevention cases. As certain prevention requires higher R&D levels, the expected profit from this action will tend to be lower than that of expected prevention, except for very low levels of unit R&D cost at which the relatively higher R&D expenditure required is more than offset by the increased output market profits from becoming a monopolist.49

7.3 R&D Competition: Accommodate or prevent activity?

As a leader can either accommodate or prevent activity by a follower, comparison of expected profits in both cases is required to determine what the leader will do. In the entry accommodation cases, (9), (22) and (45) show that expected profits are non-negative in each

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49 If R&D was costless or infinitely effective ($\gamma = 0$), expected profits from certain activity prevention would always exceed those from expected prevention. Consequently, for any level of demand volatility, there is a threshold level of unit R&D cost at which expected prevention is expected to become more profitable for any level of spillover.
case. As a leader never seeks to prevent follower activity when \( \gamma \geq 2 \), the Nash equilibrium at these unit R&D costs depends on spillovers and demand volatility as in Figure 3.

When \( \gamma = 1 \), however, both expected and certain activity-prevention are expected to be profitable for low spillovers when \( \sigma^2 = 0.05 \) (see Figure 8). Unfortunately, given the leader’s second-order R&D condition when accommodating entry in the sequential R&D game, a comparison with activity prevention at the relevant spillovers is not possible for \( \gamma = 1 \). Comparing the simultaneous R&D games of the accommodation case with the activity-prevention cases in Figure 9, when \( \beta \leq 0.1 \), both prevention cases are expected to be more profitable than accommodation, with expected prevention expected to be more profitable when \( \beta = 0.2 \). As expected profits are decreasing in demand volatility in the prevention cases, while increasing in volatility in the accommodation cases, this difference decreases as the demand shock becomes more volatile so there will be some threshold volatility level at which entry accommodation is expected to be more profitable than activity-prevention for all spillovers.

### 7.4 RJV formation (R&D co-operation)

Again setting \( \gamma = 2 \), Figure 10 shows that for almost all spillovers \( 0 \leq \beta \approx 0.94 \), the Nash equilibrium outcome is where both firms ‘wait’. For the remaining spillover levels, there are two possible Nash equilibria where both firms either ‘wait’ or ‘jump’. Comparing expected profit levels in (14) and (27), expected profits when both firms ‘wait’ exceed those from when both ‘jump’ for any positive level of demand volatility, spillover and unit R&D cost. This is due to the volatility of the demand shock having a greater effect on expected profits when both firms ‘wait’, as the output and R&D of the firms are directly affected by the demand shock, thereby ensuring that output and R&D decisions are ex-post optimal.

### 7.5 Compete in R&D or form RJV?

There are two main questions facing the firms. Firstly, when should they undertake their R&D investment and, secondly, should they compete in R&D or form a RJV? It was shown in the previous section that both firms will ‘wait’ if they form a RJV. If the firms compete in R&D, however, several Nash equilibria are possible depending on the level of spillovers and demand volatility. The question then is whether any of these Nash equilibria are expected to be more profitable than forming a RJV and ‘waiting’.

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50 Similar results follow for other admissible values of unit R&D cost.
From (17), a RJV is expected to be at least as profitable as R&D competition when both firms ‘wait’. Comparing where the firms ‘wait’ and form a RJV in (14) to where the firms ‘both jump’ and compete in R&D in (22),

\[
E\left[\pi^C\right] - E\left[\pi^N\right] \leq \frac{-8 b^2 \gamma^2 \alpha^2 (1-2\beta)^2}{(1+\beta)^2 [b\gamma - 2\theta^2 (2-\beta)(1+\beta)]} \leq 0
\]

so that ‘waiting’ and forming a RJV is expected to be more profitable for any unit R&D cost, spillover and demand volatility.

Given the above results, the interesting question is whether a firm will expect to make greater profits by ‘jumping’ and becoming a leader when the firms compete in R&D than from ‘waiting’ and forming a RJV. Comparing (14) and (45),

\[
E\left[\pi^C\right] = E\left[\pi^{LN}\right]
\]

and which profit level dominates will depend on the sign of the \{..\} term on the left hand side and \{9b\gamma-2\theta^2(2-\beta)(4-5\beta)\} on the right. When the second-order conditions are satisfied, both of these terms are positive for all spillovers when \(\gamma \geq 1\), so that

\[
E\left[\pi^C\right] < E\left[\pi^{LN}\right]
\]

(87)

For all spillovers, unit R&D cost and volatility levels, expected profits when both firms ‘wait’ and form a RJV exceed those of ‘jumping’ and being a R&D leader when the firms compete in R&D (see Figure 11 when \(\gamma = 2\)). This is due to the greater effect of demand volatility on expected profits when ‘waiting’. At low spillovers (\(\beta < \frac{1}{2}\)), the relatively high R&D of a leader when competing in R&D raises R&D costs relative to the RJV to an extent that is not offset by lower marginal production costs and, hence, expected profits are lower. At high spillovers (\(\beta > \frac{1}{2}\)), the benefits to waiting and free-riding on the relatively high R&D of the RJV partner will offset any advantage to being a leader and competing in R&D, given that R&D is a strategic complement at these spillovers.

Another interesting question is whether being a R&D leader and preventing activity by the follower is expected to be more profitable than ‘waiting’ and forming a RJV. We already know that both types of activity-prevention are only expected to be profitable when unit R&D costs and spillovers are very low (see Figures 6 and 7). In Figure 12, where \(\gamma = 1\) and \(\sigma^2 = 0.05\), both activity-prevention case have higher expected profits for very low
spillover levels. When unit R&D costs are low, the expenditure required to prevent follower activity is relatively low and the benefits of being a monopolist, either for certain or in expectation, are large enough to offset this expenditure to the extent that overall expected profits are higher. As the level of demand volatility increases, the threshold spillover at which waiting and forming a RJV is expected to become more profitable is decreasing. In Figure 13 where \( \sigma^2 = 0.3 \), certain prevention only dominates waiting and forming a RJV when \( \beta = 0 \), while the latter dominates expected prevention for all spillovers. As volatility increases, so too must the maximum level of the shock, thereby increasing the level of R&D required to prevent follower activity for certain, but also increasing the expected output of the follower in the expected prevention case which reduces the leader’s expected profits.

### 7.6 Expected welfare: output exported

While the firms’ decisions will be solely based on expected profits, from society’s point of view what matters is what maximises expected welfare. If this differs from the firms’ preference, there may be justification for government policy to increase expected welfare.

If all output is exported, then welfare is equivalent to total industry profits. We know that when \( \gamma \geq 2 \), neither activity prevention strategy is expected to be profitable. We also know that RJV formation when both firms ‘wait’ is always expected to be more profitable than competing in R&D, and also compared to a RJV when both firms ‘jump’. On the other hand, both sequential R&D cases will have a second-mover advantage, depending on spillover levels, that may lead to total profits in the competitive R&D case being greater than those of RJV formation when both firms ‘wait’.

In Figure 14 where \( \gamma = 2 \) and \( \sigma^2 = 0.05 \), total industry profits are highest for all positive spillovers when the firms wait and form a RJV. 51 In this case, any second mover advantage in the sequential R&D competition case is not sufficient to ensure greater total industry profits. Increasing R&D costs or demand volatility leads to the same outcome (see Figures 15 and 16).

On the other hand, when \( \gamma < 2 \), preventing follower activity is expected to be profitable for both types of prevention at certain spillover ranges (see Figure 8). Unfortunately, comparing activity prevention to sequential R&D competition under accommodation is not

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51 In Figures 14-23, ‘seq. R&D comp’ refers to the case where R&D levels are chosen sequentially and the firms compete in R&D. Similarly, ‘seq R&D RJV’ denotes where R&D levels are chosen sequentially and the firms form a RJV. On the other hand, exp. prev. and cert. prev. denote the expected and certain activity prevention cases, respectively.
always possible for relevant spillover levels at these unit R&D costs.\footnote{In both sequential R&D competition and sequential RJV formation under accommodation, one of the second-order conditions is not satisfied for $\beta \leq 0.2$ when $\gamma = 1$, which is the spillover range at which activity prevention is profitable (see Figure 8).} In Figure 17 where $\gamma = 1$ and $\sigma^2 = 0.05$, if $\beta \leq 0.1$, expected welfare is highest when follower activity is prevented for certain. A combination of low spillovers and unit R&D costs makes such an action profitable for the leader and, as the output market is a monopoly, total industry profit is higher. This is also consistent with the firms’ incentives (see Figure 12). When $\beta = 0.2$, total industry profit is highest when the firms wait and form a RJV. On the other hand, it is in a leader’s interests to prevent activity in expectation (see Figure 12). Given this, there may be a role for government to subsidise RJV formation in order to align private and social incentives. For all other spillovers, total industry profit, and individual firm profit, is highest when the firms wait and form a RJV.

Increasing demand volatility to 0.3 in Figure 18, waiting and RJV formation leads to highest expected welfare for all spillovers. When $\beta = 0$, however, a leader prefers to prevent activity for certain (see Figure 13) so that a subsidy for RJV formation may be required. For all positive spillovers, private and social incentives are aligned.

### 7.7 Expected welfare: domestic consumption

When output is consumed in its entirety in the economy of production, then consumer welfare will have an impact on overall welfare levels. Through the effect on output levels and, consequently, prices, if social and private incentives differ, there may again be a role for welfare improving government policy.

From (18) and (31), a RJV is expected to give at least as high an expected welfare level as R&D competition when both firms ‘jump’ or ‘wait’, so the competitive R&D cases of these games can be omitted from this analysis. Comparing all remaining cases in Figure 19 where $\gamma = 2$ and $\sigma^2 = 0.05$, when $\beta \leq 0.4$, expected welfare is highest when one firm ‘jumps’ and the firms compete in R&D. On the other hand, when $\beta \geq 0.5$, expected welfare is highest when both firms ‘wait’ and form a RJV. For relatively low spillovers, the higher R&D of the sequential investment case leads to lower marginal production costs, which has the effect of reducing output prices and increasing consumer welfare. Combined with this, the positive effect of demand volatility on the follower’s expected profit will increase expected welfare above that of when both firms wait and form a RJV. On the other hand, for relatively high spillover levels, the benefits of higher R&D levels lead to higher expected welfare when both firms ‘wait’ and form a RJV. Given this, there may be a role for government to prevent RJV formation at relatively low spillovers and co-ordinate R&D investment when the firms
compete in R&D. When the leader attempts to prevent the follower from becoming active, either in expectation or for certain, the R&D costs required to achieve these outcomes are so large that expected profits are negative (see Figures 6. and 7) even though welfare is positive through the consumer surplus effect. Such a welfare level, however, is always lower than any of the accommodation cases. Increasing unit R&D costs and demand volatility gives similar results (see Figures 20 and 21).

An interesting situation emerges when unit R&D costs are reduced to \( \gamma = 1 \) in Figure 22. When \( \beta = 0 \), expected welfare is highest when a leader prevents follower activity for certain. The relatively high R&D required to ensure this reduces marginal production costs to such a level that prices are lower and consumer surplus rises, despite the leader being a monopolist in the output market. Low unit R&D costs ensure that this action is also expected to be most profitable (see Figure 12) so that such an outcome is likely. On the other hand, at slightly higher spillovers (0.1 \( \leq \beta \leq 0.3 \)), expected welfare is highest when follower activity is expected to be prevented, though this does not always lead to the highest profit (see Figure 12). While certain prevention is expected to be most profitable at the lower end of this spillover range, over the entire range the relatively high R&D of the leader when expecting to prevent activity, leading to relatively low marginal production costs, combined with the fact that the output market may be a duopoly leads to an improved consumer surplus effect. Hence, a tax on R&D above a certain level may reduce the incentive to prevent activity for certain. On the other hand, when \( \beta = 0.3 \), RJV formation and waiting is most profitable (see Figure 12) but not socially desirable. In this case, policy may take an anti-RJV slant in the form of taxation of R&D or simply outlawing their formation.

For a narrow spillover range (0.4 \( \leq \beta \leq 0.5 \)), expected welfare is highest when firms sequentially choose their R&D and compete in R&D but this is not expected to be most profitable. Again, incentives to prevent RJV formation may be warranted. For the remaining spillovers (\( \beta \geq 0.6 \)), both firms waiting and forming a RJV is expected to be both profit and welfare maximising. The intuition for this is similar to previous cases.

Increasing the level of demand volatility in Figure 23, where \( \sigma^2 = 0.3 \), outcomes are similar for all spillovers, except that sequential R&D competition and accommodation now leads to a higher expected welfare lever for a lower spillover range (0.3 \( \leq \beta \leq 0.4 \)). As demand volatility increases, then from the firms’ perspective, the benefit of waiting and forming a RJV dominates expected prevention over a greater spillover range than before (see Figure 13).

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53 This first part of this policy is similar to D’Apremont & Jacquemin, though it may occur for a lower spillover range, while the second part may require the presence of side-payments between firms.

54 Note that the case where the firms compete in R&D and sequentially choose their R&D levels cannot be included here when \( \beta \leq 0.2 \), as the second-order conditions are not satisfied at these spillovers when \( \gamma = 1 \). A similar problem occurs for \( \beta = 0 \) when the firms form a RJV and sequentially choose R&D levels.
From a policy perspective, there is again a justification at relatively low spillovers to disincentivise RJV formation and to encourage expected activity prevention or sequential R&D competition, depending on which of these induces a greater level of expected welfare.

8. Summary and Conclusions

This paper seeks to determine the desirability of RJV formation in the presence of uncertain demand when firms benefit from R&D spillovers from rivals. The paper outlines a one-shot game where two, symmetric output-setting firms, who remain rivals in the output market, may compete or co-operate in R&D at the pre-output stage, but may also choose to undertake their R&D investment before any demand uncertainty is resolved, thereby possibly being a R&D leader. By becoming a R&D leader, R&D investment may be chosen to ensure that the follower does not become active in the industry, either in expectation or for certain.

If the firms compete in R&D, and a R&D leader accommodates the follower, then for given levels of unit R&D costs, multiple Nash equilibria are possible, depending on the level of spillovers and demand volatility. Over a large range of spillover and volatility levels, both firms waiting until the demand uncertainty is resolved is the unique Nash equilibrium outcome. At very low levels of spillovers and demand volatility, however, both firms ‘jumping’ and investing in R&D before the demand uncertainty is resolved is the unique Nash equilibrium outcome. For slightly higher volatility levels, two Nash equilibria are possible where both firms simultaneously choose their R&D either before or after the demand uncertainty is resolved. Which equilibrium leads to greater expected profits for the firms will depend on the level of spillovers. At relatively high spillovers but low volatility levels, two Nash equilibria are possible where the firms sequentially choose their R&D levels.

When forming a RJV, both firms waiting until the demand uncertainty is resolved before undertaking their R&D investment is again the unique Nash equilibrium for most spillover and volatility levels. At very high spillover levels, however, two Nash equilibria can occur where the firms simultaneously choose their R&D, either before or after the demand shock occurs, though the latter is more profitable for the firms.

If a R&D leader seeks to prevent any activity by the follower, either in expectation or for certain, such an action is only expected to be profitable at very low levels of spillovers and unit R&D costs and exceeds that of simultaneous R&D investment.

If both firms wait or jump, then forming a RJV is expected to be at least as profitable as R&D competition. The most interesting question is whether R&D leadership when competing in R&D is expected to be more profitable than waiting and forming a RJV? If the leader accommodates the follower, the latter is the weakly preferred course of action of the firms for all levels of demand uncertainty. On the other hand, both activity-prevention cases
are expected to be more profitable than a ‘waiting’ RJV when unit R&D costs and spillovers are very low, with the difference negatively related to demand volatility.

If maximising expected welfare is the objective of the government, this may require a different action by the firms to that which is expected to be profit maximising. If output is exported in its entirety, there may be a role for government to subsidise RJV formation for relatively low levels of unit R&D costs, spillovers and demand volatility. On the other hand, if all output is consumed domestically, government policy could take the form of preventing RJV formation, taxing R&D expenditure that is incurred before any uncertainty is resolved or co-ordinating competitive R&D investment in conjunction with overseeing a system of side-payments to ensure that total industry profits are equally shared. Which policy is required depends on unit R&D costs, spillovers and demand volatility.

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Figure 1 – RJV formation ($\beta < \frac{1}{2}$)
Figure 2: Expected RJV profits, leader v follower ($\theta = b = 1$)

Expected profit vs spillover ($\beta$)

- $E(\pi_u^{LC}) > E(\pi_u^{RC})$
- $E(\pi_u^{LC}) < E(\pi_u^{RC})$

Leader RJV SOC vs equal profit

$\beta = 5/7$

Figure 3 - Nash equilibria, R&D comp. ($\gamma = 2$)

- (J, J) & (W, W)
- (W, W)
- (J, W) & (W, J)
- (J, J)

$\phi$ vs $\psi$ vs spillover ($\beta$)

$E(\pi_u^{LC}) < E(\pi_u^{RC})$
Figure 4 - Leader's expected profits - expected v certain prevention

\((\gamma = 2)\)

\[ \sigma < 0.577 \]

Figure 5 - Leader's expected profits - expected v certain prevention

\((\gamma = 1)\)

\[ \sigma > -0.577 \]

\[ \sigma_{\text{max}} \]
Figure 6: Leader’s expected profits - expected prevention ($\sigma^2 = 0.05$)

Figure 7: Leader’s expected profits - certain prevention ($\sigma^2 = 0.05$)
Figure 8: Leader’s expected profit - expected vs certain prevention ($\gamma = 1$, $\sigma^2 = 0.05$)

Figure 9: Leader’s expected profits, R&D comp ($\gamma = 1$, $\sigma^2 = 0.05$)
Figure 10: Nash equilibrium, RJV ($\gamma = 2$)

Figure 11: R&D comp leader v 'waiting' RJV ($\gamma = 2$)
Figure 12: Expected profit - waiting RJV v prevention ($\gamma = 1, \sigma^2 = 0.05$)

Figure 13: Expected profits - waiting RJV v prevention ($\gamma = 1, \sigma^2 = 0.3$)
Figure 14: Expected Welfare - output exported ($\gamma = 2, \sigma^2 = 0.05$)

Figure 15: Expected Welfare - output exported ($\gamma = 5, \sigma^2 = 0.05$)
Figure 16: Expected Welfare - output exported ($\gamma = 2, \sigma^2 = 0.3$)

Figure 17: Expected welfare - output exported ($\gamma = 1, \sigma^2 = 0.05$)
Figure 18: Expected welfare - output exported ($\gamma = 1$, $\sigma^2 = 0.3$)

Figure 19: Expected Welfare - domestic cons ($\gamma = 2$, $\sigma^2 = 0.05$)
Figure 20: Expected welfare - domestic cons ($\gamma = 5$, $\sigma^2 = 0.05$)

Figure 21: Expected welfare - domestic cons ($\gamma = 2$, $\sigma^2 = 0.3$)
Figure 22: Expected welfare - domestic cons ($\gamma = 1$, $\sigma^2 = 0.05$)

Figure 23: Expected Welfare - domestic cons ($\gamma = 1$, $\sigma^2 = 0.3$)