Non-linearities in drip water hydrology: an example from Stump Cross Caverns, Yorkshire

Andy Baker\textsuperscript{a,\ast}, Chris Brunsdon\textsuperscript{b}

\textsuperscript{a}Centre for Land Use and Water Resources Research (CLUWR), University of Newcastle-upon-Tyne, Newcastle NE1 7RU, UK
\textsuperscript{b}Department of Geography, University of Newcastle-upon-Tyne, Newcastle NE1 7RU, UK

Received 2 September 2002; accepted 24 January 2003

Abstract

Drip rate data have been collected at 15 min intervals at six locations in Stump Cross Caverns, N England, since 1998. The different drip sites cover a wide range of drip rates from \(\sim 2\) drips/s to \(2\) drips/h, and in general the variability of drip rate increases with mean drip rate. In our continuous data sampling we observe rapid discharge increases which appear to be synchronous between drips sites, and which can be explained by flow switching of the water overlying the cave during times of high infiltration rate, such as intense rain storms or rapid snowmelt. A test for non-linearity (White test) in the drip series provides very strong evidence that many of the drip sequences are non-linear. We conclude that at our drip sites there is a non-linear input (weather) and non-linearities within the karst system leading to non-linear dripping, which is independent of drip rate. Our results have implications for stalagmite palaeoclimatology, where such widespread non-linearities have not been taken account of.

Keywords: Drip discharge; Data logger; Speleothems; Non-linearities; Palaeoclimate

1. Introduction

Recent research using automatic data loggers of karst spring discharge, with measurements being taken at 15 min and daily intervals on springs draining 10-100 km\(^2\) catchments and with discharges of 0.4-2.3 m\(^3\) s\(^{-1}\), has shown that the karst system is inherently non-linear, due to its spatial, dynamical and physical heterogeneity and that it can only be modeled by non-linear models (Labat et al., 2000a,b; 2002). For example, Labat et al. (2000a) model spring discharge and rainfall using linear black box methods, and demonstrate that linear models such as stochastic and Fourier models do not reflect the hydrological behaviour of the springs at flow extremes due to non-stationary and non-linear nature of the system. This is due to both karst physical heterogeneity (in space) and dynamic variability (in time) together with non-Gaussian inputs (rain) and outputs (discharge). In a later paper, multifractal analysis on the same spring records show scale dependant behaviour, with different multifractal processes at each sampling site (Labat et al., 2002). At daily sample interval, the springs have similar multifractal properties that are invariant...
between basins. At 30 min resolution, two multi-fractal sub-processes are evident, suggesting that multiscale non-linear models are necessary to model the hydrology of these springs. Implications of this research include the fact that linear models of water flow in the karst system poorly predict flow extremes (high and low discharge), which is important for contaminant flows and flood warning. Another implication of the research is the necessity for high temporal resolution automatically logged data to better understand the karst spring discharge regimes.

A key question raised by the work on small karst catchments is whether the same non-linearity is observed at the lower discharge drip-waters observed in cave systems? This requires an understanding of the karst aquifer, which can be divided up into a number of components:

1. The ground surface and soil. This is the interface between surface precipitation (rain, snow, sleet, hail) and the ground surface, and the soil, together with its interactions with any vegetation, is important as a store of water and has an important role in the process of evapotranspiration;

2. The surface few meters of the limestone, often called the epikarst. This zone, due to enhanced dissolution and weathering, is often more permeable, and is characterized by fractures and fissures of different widths and sizes which will have different responses to infiltration from the surface. The epikarst is also within the unsaturated zone.

3. The lower limestone which is permanently saturated.

As well as these three components, it should be recognized that different karst aquifers have differing extents of karstification, depending on their age, geology, etc., and that they will grade from being dominated by diffuse flow at one extreme to being dominated by conduit or fissure flow at the other. The key here is that the work on karst spring discharges (Labat et al 2000a,b; 2002) potentially involves all three components, whereas drip-waters in caves are by definition found in the unsaturated zone and therefore only affected by processes in the first two components. To date, there are few published studies using high temporal resolution drip loggers in caves, with most research programmes use manual sampling over 'PhD time' of 1–3 years (Pitty, 1966; Baker et al., 1997, 1999; Genty et al., 2001). This could lead to the development of over-simplistic models, for example that of Smart and Friedrich (1987) which failed to recognize the presence of high flow variability at low discharge. Recent research by Genty and Deflandre (1998) which did use high temporal resolution data (10 min logging for 5 years in Grotte Père Noël, Belgium) suggested that although inter-annual drip rate (e.g. total discharge per year) correlated well with annual water excess (rainfall-evaporation), with strong seasonal variations related to seasonal water excess:

1. there was a superimposed 1–7 days variation in drip rate due to two phase flow (air and water) in the fissure, leading to a correlation with surface air pressure variation. The sensitivity to air pressure was greatest at intermediate discharges of 50–400 drips/10 min.

2. there was unstable hydrology with flow switches (a change in discharge in less than 2 h) or periods of unstable discharge above a threshold of 24 drips/min.

Therefore non-linear and possibly chaotic behaviour has been reported at the scale of individual drip waters, but this only for one drip, and therefore further research is needed to investigate how typical this behaviour is in drip waters.

Understanding any non-linearities in drip hydrology is of crucial importance as there is a large body of research which attempts to utilize cave stalagmites as an archive of past climate and environmental change (Lauritzen and Lundberg, 1999; Proctor et al., 2000, 2002; Fairchild et al., 2001; McDermott et al., 2001), increasingly at annual or sub-annual resolution. For example, drip waters carry geochemical information from rainwater (for example δ18O, δD, trace elements), which then interacts with the soil component described above (that include organic matter and elements such as calcium, magnesium, strontium and phosphorous) and geology (where dissolution and reprecipitation of calcium, magnesium, and strontium can all occur), before finally being preserved in stalagmites. Therefore drip hydrology directly affects the majority of the geochemical signals preserved in
stalagmites, due to the complex interactions between the water and the soil and bedrock. Any nonlinearities in drip hydrology, including flow switching between different water sources or flow routes, nonlinear responses to surface rainfall, evapotranspiration and/or water table changes, and mixing between event water and pre-event water, could all lead to the interpretation of extreme climate and environmental changes in stalagmite proxies when indeed none have occurred, as technological advances such as laser-ablation $\delta^{18}$O (McDermott et al., 2001) and SIMS trace element analyses (Fairchild et al., 2001) have lead to $10^9$–$10^3 \mu$m analyses, suggesting that it will soon be possible to resolve daily hydrological events in fast growing stalagmites.

2. Site description and sampling methodology

Stump Cross Caverns is a upland cave situated in a high fractured inlier of Carboniferous limestone in N England. The site is overlain by both thin peat and gley soils, with a heather-bog-upland grass vegetation, with limited sheep grazing and also some reforestation. Rainfall has little seasonal variability, with an annual mean of 1500 mm. Mean annual temperature is $\sim$7 °C, with summer temperatures high enough for a soil moisture deficit to form. Rainfall and soil moisture data (calculated from the Thornthwaite equation; Thornthwaite, 1955) for the study period are shown in Fig. 1. Local climate data was obtained from a manual weather station in the nearby town of Pateley Bridge, 5 km to the east of the study site.

Drip water sample locations are shown in Fig. 2; all are on the upper ‘show cave’ level of the caverns, about 15 m below the surface. The cave is heavily decorated with speleothems, and six sensors have been deployed since 1998 at drip sites that were chosen for their wide range of discharges. Two sites (stalagmite 2 and stalagmite 4) have been logged continuously since April 1998. Logging at stalagmite 1 was stopped due to construction of a new show cave section in late 1999 and moved to a new site, flowstone 7. At the same time, loggers 3 and 5 were deployed for the first time. Small data gaps occur at several sites due to sensor failures. Several of the logged sites (sites 1, 3 and 5) have candlestick shaped stalagmites that would be conventionally thought suitable for climate and environmental reconstruction. Site 2 has a slightly broader stalagmite that is boss

---

Fig. 1. (Top) Measured rainfall from Pateley Bridge; (Bottom) calculated hydrologically effective precipitation.
shaped, site 4 is a stalactite drip that supplies a water pool and site 7 a drip fed flowstone. Data logging works on a vibrating drum principle, every time a drip falls onto the drum it vibrates and this signal is converted into an electrical pulse. This is transmitted through wiring to a Campbell Scientific Data Logger that counts up all these electrical pulses and downloads this data to computer every 15 min. All data is displayed real-time in the caverns.

3. Results

Data logger results are presented in Fig. 3 and descriptive statistics in Table 1. Drip volume was not measured, so investigation of any drip volume— discharge relationship as observed by Genty and Deflandre (1998) is not possible. The mean values demonstrate that stalagmites 1 and 2, and flowstone 7 are the fastest drippers, and that sites 3, 4 and 5 drip at a very much slower rate. In terms of standard deviation, flowstone 7 shows the most variation. It is also interesting to note that despite having the highest drip rate, stalagmite 1 does not have the largest standard deviation—it exhibits less variation than flowstone 7. Also, stalagmite 2 has a relatively high standard deviation in comparison to its mean drip rate—this higher level of variability is also evident in the extremely high maximum drip rate, and is reflected in the high degree of skewness for that stalagmite. Certainly in this case, there is strong evidence for a non-Gaussian distribution. Finally the number of unique values of drip rate is tabulated for each site—this is mainly to identify the level of discreteness in each log. The data for sites 3, 4 and 5 show a notable degree of discreteness, with drip rates only taking 10, 11 or 4 different values. This could lead to problems in testing their statistical properties—some apparent effects may be an artefact of the discreteness of the recording process rather than a genuine property of the dripping process.

3.1. Description of drip hydrology

Stalagmite 1 has a mean drip rate of approximately 6 s/drip. Dripping continues all year long, despite a soil moisture deficit occurring each summer (Fig. 1), suggesting that there must be a significant storage flow component to this drip such that drip rate is maintained all year. Drip rates increase rapidly (within days) after autumn recharge (for example, after day numbers 300 and 620) and decreases to a minimum discharge in summer, showing that there is also a (probable fissure flow) component which responds to periods of hydrologically effective precipitation (Fig. 1). Finally, Fig. 3 also demonstrates that there is a high frequency variability of drip rates onto this stalagmite, with drip rates varying by \(^ {100}\) drips/h over a 5–10 day period. A similar observation was made by Genty and Deflandre (1998) in their study of a stalactite drip and explained as being generated by either a 'shut-off faucet' process due to the rock formation stress, or to a change in the two-phases flow component proportions (air/water). We also see a negative correlation with these high frequency drip rates variations and air pressure (data not shown), suggesting the same processes are occurring here.
Stalagmite 2 has similar mean drip rate to stalagmite 1, again with dripping maintained all year despite the soil moisture deficit in summer, suggesting a storage flow component to this drip. However the drip has a greater variability of flow with the drip slowing significantly in summer and on at least four occasions (27/10/98, 4/4/00, 19/9/00, and 30/10/00) this stalagmite demonstrates extremely rapid increases in drip discharge. Drip rate increases from pre-event flow to drip rate maxima occurs within three hours, and reach a maximum of 1000–4000 drips/h, suggesting that under some conditions a very rapid flow component is present.
Sample site 7 is a drip that feeds a flowstone within 5 m of stalagmite 1. This site has a high mean drip rate, a high variability of drip rates and responds rapidly to surface rainfall similar to stalagmites 1 and 2. The range of drip rates is such that dripping stops during most summers, suggesting that for this drip the storage flow component is smaller than for stalagmites 1 and 2. A rapid drip rate increase on the same occasions listed for stalagmite 2 is observed for the period that this drip logger was active.

Stalagmite 3 has a much slower mean drip rate than stalagmites 1 and 2, and its range and variability of drip rate are both correspondingly lower. This could be explained by a drip source that has a greater proportion of stored water component than the previous samples. However, on occasion the drip rate still shows a rapid response, with short-lived drip rate peaks on 4 April and 18 September 2000 that occur at the same time as rapid drip rates at stalagmite 2.

Stalagmite 4 drips every 1−5 min. The seasonal drip rate increases lag the autumn increase in hydrologically effective precipitation by about two months. The slow drip rate with low variability and lagged response suggests that the stored water component of flow is the dominant component, and that over the period of data collection no rapid, fissure flow occurred.

Finally, stalagmite 5 drips at far the slowest rate, with a mean drip rate of 2−4 drips/h. This drip has very little variability with any seasonal trend lagging any increase in hydrologically effective precipitation by ~180 days.

The different drip sites therefore cover a wide spectrum of drip rates from ~2 drips/s to 2 drips/h, and in general the variability of drip rate increases with mean drip rate as observed elsewhere (Smart and Friedrich, 1987; Baker et al., 1997). Of particular interest in our continuous data sampling is the presence of rapid discharge increases, which appear to be synchronous between drips sites. These are shown by asterisks in Fig. 3. Their frequency decreases with decreasing discharge; for example for the same time interval three events are observed at drip 2 compared to two at drip 3. The rapid drip rate increases are visible even down to drip rates of 40 drips/h (stalagmite 3), a drip which otherwise shows typical storage flow dominated behaviour. Such rapid drip rate increases can be explained by flow switching of the water overlying the cave during times of high infiltration rate: for example the activation of overflow routes or switching between two competing flow routes. Given that such ‘event’ water might contain a different isotopic, pollutant or geochemical signature than ‘non-event’ drip water, which might become preserved in the stalagmite calcite or cause aquifer pollution, it is necessary to investigate the climatic forcing of these rapid drip events.

3.2. Rapid drip increases and their climate forcing

Comparison of the timing of the four most apparent drip rate increases (27/10/98, 4/4/00, 19/9/00, and 30/10/00) and the prevailing climate (Fig. 1) demonstrates that three of the events were generated within the top four mean daily rainfall totals (>43 mm). These were all associated with disturbed and stormy weather. However, one of the four highest mean daily rainfall days did not lead to a rapid drip rate event. This was the 5 November 2000 when 47.8 mm of rainfall was recorded in the preceding 24 h. This was after 3 days of >39 mm in the previous week, including the 30/31 October which did generate a drip event. The synoptic conditions related to each event
are summarized from monthly weather logs of the Royal Meteorological Society:

27 October 1998. Very disturbed weather with deep depressions and active fronts with frequent and prolonged falls of rain. 47.3 mm of rain was recorded in Pateley Bridge (5 km to the east) in the 24 h to 9 a.m. on the 27 October. The nearby town of Skipton was flooded as the River Aire reached its highest level for 50 years.

19/20 September 2000. Very wet with slow-moving fronts in a southerly airflow bringing heavy prolonged downpours. 44.4 mm of rain was recorded in Pateley Bridge in the 24 h to 9 a.m. on the 20 October. Regional rivers were on flood alert for several days.

30/31 October 2000. The month of October 2000 was the wettest since 1903. On the 30/31, the weather was very disturbed with severe south-westerly gales and torrential downpours. On the 30 an intense secondary depression crossed England, leading to the lowest October atmospheric pressure recorded in England. Extensive structural damage and flooding occurred with the nearby River Aire flooding and houses evacuated in Skipton. In Pateley Bridge 43.7 mm of rain was recorded in the 24 h to 9 a.m. on the 31st October.

Additionally, one event was associated with low rainfall totals but instead caused by infiltration after a rapid thaw of snow cover:

4 April 2000. Previously lying snow underwent a rapid thaw on the 4th as a westerly depression tracked through N France and English Channel. Only 0.3 mm of rainfall fell on the day, but snowmelt lead to flooding in many English Midland rivers.

Analysis of the climate data shows that rapid drip events are generated by climate events that generate rapid infiltration, and that these can be both due to high daily rainfall totals and rapid snowmelt. In addition, not all days of high rainfall totals generated a rapid drip event; unfortunately there is no reliable hourly rainfall data for the site for us to investigate if this is due to rainfall intensity being a threshold rather than rainfall amount. However, as described in the introduction, although there may be a regular climate forcing in terms of seasonal variations in hydrologically effective precipitation, which drives the low frequency drip variations for most of the sites, the distribution of extreme infiltration events which force the rapid drip events is unlikely to be Gaussian. Furthermore in some cases the system may not be linearly responsive. Further insight into the drip events is possible through statistical analysis.

3.3. Statistical analysis

3.3.1. Autocorrelation

Undetrended autocorrelation plots are presented in Fig. 4 for the drip sites, and show that most samples have a long period of statistically significant autocorrelation for 4000–6000 data points (1000–1250 h or 42–52 days). This autocorrelation is due to the seasonal trend in the data of seasonal changes in drip rate due to summer soil moisture deficit and winter hydrologically effective precipitation. Drips 1 and 5 are different from the rest. Drip 5 has very little seasonal variation in drip rate and an essentially invariant drip, and therefore has no autocorrelation. Drip 1 in contrast has very little ‘memory’ of its preceding drip rate, with no a strong autocorrelation for the first 200 data points (~2 days), followed by alternating positive and negative correlations. This lack of memory is generated by the 5–10 day periodicity in drip rate of this stalagmite, forced by air pressure variations as described earlier. These variations of ~100 drips/h are of a similar order of magnitude as the seasonal trend in mean drip rate.

Detrended autocorrelation graphs (Fig. 4, bottom) remove the seasonal trend in the data, and drips 1, 2 and 7 now all have interesting autocorrelation patterns. Drip 1 maintains a similar autocorrelation plot for the reasons described above-detrrending effectively has no effect on this data set. Drips 2 and 7 have periods of statistically significant autocorrelation at about 4000 data points (~42 days; stalagmite 2) and 1000 and 2000 data points (~21 and ~12 days; flowstone 7). It is likely that these are an artefact of the extreme infiltration events which cause transient bursts of very high drip rates at certain times; when each of the high drip rate events is removed and the autocorrelation rerun, these periods of statistically significant autocorrelation move or disappear altogether.

3.3.2. Cross-autocorrelation

In Fig. 5, cross-correlations between some of the drip rates are shown. Here, the only relationships
Fig. 4. Undetrended (top) and detrended (bottom) autocorrelation graphs.
illustrated are between the ‘fast drippers’, as these have the largest number of unique values. Each graph shows the cross-correlation for the pair of drips shown in the title, as well as 95% confidence levels. The number underneath is the number of observations used—in order to evaluate the cross correlation function (CCF) it is necessary to have a run of observations common to both samples. For all drip pairs, there are many periods of statistically significant cross correlations, but which have no linear relationship with lag number, suggesting that there is no simple linear relationship between the drip sites. Once again, two of the charts associated with stalagmite 2 show a notable effect at the 4000 mark—again this is likely to be a consequence of the transient bursts of very

<table>
<thead>
<tr>
<th>Site</th>
<th>Stall</th>
<th>Stal2</th>
<th>Flow7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-squared (raw)</td>
<td>0.25</td>
<td><strong>611.25</strong></td>
<td>1.08</td>
</tr>
<tr>
<td>Chi-squared (detrended)</td>
<td><strong>63.1</strong></td>
<td>777.5</td>
<td><strong>1064.5</strong></td>
</tr>
</tbody>
</table>
high drip rates occurring during high infiltration events.

3.3.3. White neural net test

Having considered the auto and cross correlations, it seems important to investigate non-linearity in the drip rates. One non-linearity test is the White Neural Net test for non-linearity (Lee et al., 1993). This is a statistical test for non-linearity in a recursive time series. If there is a relationship of the form $t(n + 1) = f(t(n)) + e$ (where $e$ is a random error term, $t(n)$ and $t(n + 1)$ are values of a time series at times $n$ and $n + 1$; respectively, and $f$ is a function), then the White test tests the hypothesis that $f$ is a linear function, i.e. $f(x) = ax + b$.

We applied the White test to all the drip logs that had enough discrete values to permit testing; sites 1, 2 and 7; and we applied the test to both the raw and detrended data. The results of applying the White test is listed in Table 2; the test statistic for a White test is chi-squared with two degrees of freedom under the null hypothesis that $f(x) = ax + b$ so that high values of this statistic are indicative of non-linearity. The 99.9% point of this distribution is 13.82. Note that in the raw data only stalagmite 2 is classed as non-linear. For flowstone 7 the series is linear, probably due to the strong seasonal signal, and for stalagmite 1, possibly due to the regular 5–10 days drip rate variability caused by the two-phase flow. When the detrended data is analysed (i.e. the effect of seasonality is removed), all three series are non-linear, demonstrating that the non-linear high frequency drip rate variations are superimposed on either linear or non-linear seasonal drip rate trends. The cause of the non-linearity observed in the detrended

![Fig. 6. T vs t + 1 plot for stalagmite 2.](image)
series could be due to non-linear infiltration inputs, and/or non-linear processes within the karst aquifer; the two can not be distinguished statistically.

3.3.4. t vs t+1 plots

In the previous section, evidence was found that the some of the drip sequences were non-linear. A useful means of investigating non-linearity is a phase plot—where the observation at time \( n \) is plotted against the observation at time \( n + 1 \): This enables the relationship between \( t(n) \) and \( t(n + 1) \) discussed in the previous section to be investigated—and in more complex non-linear models may still provide some clues about the properties of the model. Here we plot the results for stalagmite 2, which was demonstrated to be non-linear in both the raw and detrended series, in Fig. 6. The plot appears to be roughly linear in the lower left hand section, but takes on a very different characteristic in the remainder of the plot. There is a distinct L-shaped section in the plot, also observed by Genty and Deflandre (1998), and indicative of non-linear drip behaviour. Similar structures were not as apparent in phase plots of stalagmite 1 and flowstone 7.

4. Conclusions

There is a non-linear input (weather) and non-linearities within the karst system leading to non-linear dripping. Our results confirm that the non-linear discharge from karst springs observed at discharges of 0.4–2.3 m\(^3\) s\(^{-1}\) by Labat et al. (2000a,b, 2002) are also observed at some low discharge (< 10\(^{-5}\) m\(^3\) s\(^{-1}\)) speleothem drips at Stump Cross Caverns. We also observe two-phase flow in one drip (stalagmite 1), as observed by Genty and Deflandre (1998) as a 'wiggle regime', but not the 'unstable regime' observed by the same authors. For our sites the presence of non-linear dripping does not correlate with mean drip rate; stalagmite 2, which as the most non-linear drip series has the third highest mean drip rate of the series investigated here. Our results suggest that further research is needed. In particular, drip water sampling during non-linear drip events would be of interest to see if they would leave a distinctive geochemical or isotopic signature in the stalagmites, which could be used to reconstruct extreme infiltration events in the past. At our site, stalagmite drip sites 1 and 3 have regular candlestick stalagmites forming beneath them, ones that might be selected for palaeoenvironmental reconstruction and previously assumed to have a relatively linear response to climate and drip rate. However, Fig. 3 clearly shows rapid drip rate events that might contain unique distinctive geochemical or isotopic signatures. Further automatic logger data is required from other cave sites in order to put the Stump Cross data into context; is it typical or atypical of speleothem drip regimes? Caves at different depths, in differing geology and structure, and in different climatic regions would all be of interest in the investigation of the importance of non-linear flows in cave drips. In particular, research at sites where the infiltration input is linear (sites with a strong and regular seasonality of rainfall) would enable the identification of the relative importance of input and aquifer as sources of non-linearity. Finally, it would be interest to see to what extent the drip time series presented here can be theoretically modelled, given that the latter are based on linear algorithms.

Acknowledgements

Geoff Workman, Harry Long, Gordon Hanley and his family for the huge amount of hours spent installing the cabling and maintaining the loggers in the absence of A.B. Bill Wade at the north east regional group of the Royal Meteorological Society for Pateley Bridge meteorological data. Andy Bartram at the Department of Geography, University of Exeter, designed the drip sensors. Dr Robin Cullingford, formerly of the University of Exeter undertook computer programming. Dr Mike Simms of the Ulster Museum designed display boards at Stump Cross where real-time data-logging can be observed. Logger maintenance and field support occurred with the help of George Kania, Jo Baker, Laura Burns, Jim Gourlay, and Martin Springmore. The project was part-funded by a grant to Baker, Cullingford, Simms, and Stump Cross Caverns from the Millennium Commission, The Royal Society and the British Association in 1997.
References


