Computationally efficient sequential learning algorithms for direct link resource-allocating networks

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Abstract

Computationally efficient sequential learning algorithms are developed for direct-link resource-allocating networks (DRANs). These are achieved by decomposing existing recursive training algorithms on a layer by layer and neuron by neuron basis. This allows network weights to be updated in an efficient parallel manner and facilitates the implementation of minimal update extensions that yield a significant reduction in computation load per iteration compared to existing sequential learning methods employed in resource-allocation network (RAN) and minimal RAN (MRAN) approaches. The new algorithms, which also incorporate a pruning strategy to control network growth, are evaluated on three different system identification benchmark problems and shown to outperform existing methods both in terms of training error convergence and computational efficiency.

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1. Introduction

While radial basis functions (RBF) have been used for many years for high dimensional interpolation, it was not until 1988 that they were introduced by Broomhead and Lowe [6] as the basis for a neural network architecture. They operate by generating a weighted linear combination of a set of basis functions (normally Gaussian) defined on the input space to give an overall input-output mapping:

\[ y_k = f(x_k) = \sum_{i=1}^{m} h_i \phi_i(x_k; c_i, \sigma_i) + b, \]

(1)

\[ \phi_i(x_k; c_i, \sigma_i) = \exp \left( -\frac{||x_k - c_i||^2}{\sigma_i^2} \right). \]

(2)

Here \( h_i \) are the linear output layer weights, \( c_i \) and \( \sigma_i \) are the centres and widths of the Gaussian basis functions, \( b \) is an optional bias weight and \( m \) is the number of neurons in the network. Practically RBF networks have a number of advantages over the well-known multilayer perceptron (MLP) with regard to training, locality of approximation and transparency. In particular, RBF networks with localised basis functions learn information at one operating point of a nonlinear process without degrading information assimilated at other operating regimes [16].

The main concern when using RBF networks is how to select the centres for the basis functions so that they provide adequate support for the mapping. The traditional approach of placing them on a uniform grid or at the location of each input vector from a subset of the input-data is generally not practical due to the curse of dimensionality. Prior selection of RBF centres with the aid of clustering techniques is possible if representative training data is available a priori. However, even if such data is available there is no guarantee that number and placement of centres will be appropriate if the plant operating point and/or dynamics change significantly with time.

Resource-allocating networks (RAN) were introduced by Platt [17] as a solution to these problems. RANs are simply RBFs in which recursive training of parameters is combined with a procedure for recursively adding centres. In [17] RAN weights were updated using stochastic gradient descent. Kadirkamanathan and Niranjan [10] modified RAN by developing a second order recursive method, known as RAN-EKF (extended Kalman filter) and described it as sequential learning. Yingwei et al. [19] extended the work on RAN-EKF by including a pruning strategy to obtain a more parsimonious RBF network and called it minimal RAN (MRAN).

In this paper two approaches to reduce the computational and memory requirements of sequential learning algorithms are considered. The first involves decomposing training on a layer by layer and neuron by neuron basis and is achieved by neglecting inter neuron weight interactions. This leads to a significant reduction in memory requirements for algorithms that employ cost function curvature information in their weight updates as the approximation of the Hessian matrix (or its
Network decomposition was first introduced by Jankowski and Kadirkamanathan [8] in connection with the weight covariance matrix in the RAN-EKF algorithm and more recently by Asirvadam et al., [1,3] in connection with a recursive Levenberg–Marquardt algorithm proposed by Ngia and Sjöberg [14].

The second approach to improving efficiency is the adoption of a minimal weight update policy. Here the basic idea is that, rather than updating the entire set of weights at each iteration, only the weights associated with the neuron (or neurons) closest to the current input in the network, known as the winner neuron(s) are updated. Junge and Unbehauen [9] did pioneering work on minimal update approaches and a similar methodology has also previously been applied by Asirvadam et al. [2]. Li et al. [11] introduced a minimal update policy for MRANs and showed it to be superior to Junge and Unbehauen’s [9] minimal update approach.

The main contribution of this paper is to combine these two approaches to reducing complexity in order to maximize the computational efficiency and performance of sequential learning of radial basis function models, thus increasing their viability for real-time identification of nonlinear processes. Extended-Kalman filter (EKF) and recursive Levenberg-Marquardt (RLM) implementations are developed for a general class of RAN referred to as direct-link RANs (DRAN) or direct-link RBFs (DRBF). DRANs are simply conventional RBFs, as defined by eq. (1), augmented by a linear input–output mapping, that is

\[ y_k = \sum_{i=1}^{m} h_i \phi(x_k; c_i, \sigma_i) + b^T \bar{x}_k. \]  

(3)

The bias term is included as weight \( b_0 \) in vector \( b \).

\[ b = [b_0 \ldots b_n]^T \text{ with } \bar{x}_k = [1 \ x_k^T]^T. \]  

(4)

Parameter \( n_i \) is the number of inputs fed to the network. DRBFs provide a natural extension from linear-to-nonlinear modelling and facilitate the inclusion of a priori linear modelling experience. They are particularly beneficial in sequential learning since the direct link (linear) term has global support and thus provides a degree of generalisation even when the network is in its infancy.

The remainder of the paper is organised as follows. Section 2 describes sequential learning for RANs and the pruning strategy developed by Yingwei et al. [19]. Section 3 introduces decomposed EKF and RLM for DRANs while Section 4 describes the minimal update alternatives. Simulation results evaluating the new training schemes are presented in Section 5 and finally Section 6 concludes the paper.

2. Sequential learning algorithms

Sequential learning algorithms differ from classical RBF training rules in that they combine new centre allocation with weight updating in one routine. The basic
algorithm structure is as follows:

\[
y_k = \sum_{i=1}^{m} h_i \exp \left( -\frac{\|x_k - c_i\|^2}{\sigma_i^2} \right). \tag{5}
\]

Let \((x_k, y_k)\) be a new data point to be fitted by the RBF network in (5). The sequential learning growth criteria (which is based on [16]) for the network are

\[
d_{nc} < 2\sigma_{nc} \tag{6}
\]

and

\[
\sum_{j=k-(M-1)}^{k} \frac{|e_j|}{M} > \bar{e}_{\min}, \quad e_j = d_j - y_j. \tag{7}
\]

In (6) \(d_{nc}\), defined as,

\[
d_{nc} = \min_i \|x_k - c_i\|, \quad i = 1, 2, \ldots, m, \tag{8}
\]

is the distance between input vector \(x_k\) and the centre of the nearest hidden neuron, \(\phi_{nc}\) while \(\sigma_{nc}\) is the width of the nearest neuron. The desired output corresponding to input \(x_j\) is given by \(d_j\) while the scalar \(\alpha\) (determines the Gaussian locality range) is usually 1.0. The error threshold parameter, \(\bar{e}_{\min}\), in (7) is computed as

\[
\bar{e}_{\min} = \max(\bar{e}_{\min}, \gamma \bar{e}_{\max}). \tag{9}
\]

An average moving-window instantaneous error is used as a novel criterion instead of the instantaneous error \(e_k\) as the latter is vulnerable to outliers [12]. Scalars \(e_{\min}\), \(e_{\max}\) and \(\gamma < 1\) are the user defined parameters.

If either (or both) criteria in (6) and (7) are not satisfied then all the network parameters are adapted to fit the new point using a recursive Gauss–Newton method, also known as the EKF algorithm [10]. This can be described as follows:

\[
w_k = [c_1^T, \sigma_1, h_1, \ldots, c_m^T, \sigma_m, h_m]^T_{N_w \times 1}, \tag{10}
\]

\[
w_{k+1} = w_k + P_k \psi_k e_k, \tag{11}
\]

\[
e_k = d_k - y_k, \tag{12}
\]

\[
\psi_k = \frac{\partial}{\partial w_k} y(x_k, w_k). \tag{13}
\]

\[
P_k = P_{k-1} - \frac{P_{k-1} \psi_k \psi_k^T P_{k-1}}{1 + \psi_k^T P_k \psi_k}. \tag{14}
\]

Here \(N_w\) is the total number of parameters in the network. On the other hand if the growth criteria in (6) and (7) are satisfied then a new Gaussian basis function is assigned as follows:

\[
c_{m+1} = x_k, \quad h_{m+1} = |e_k|, \quad \sigma_{m+1} = \beta \frac{d_{nc}}{2}. \tag{15}
\]
\[ P_k = \begin{bmatrix} P_{k-1} & 0 \\ 0 & I_{n_w \times n_w} \end{bmatrix}. \] (16)

The scalar \( \beta \) is a user defined parameter (usually set to 1.0) which determines the degree of overlap between neurons and the vector dimension \( n_w \) is equal to the number of new parameters arising from the inclusion of the new Gaussian basis function. This form of sequential learning is known as RAN-EKF [10]. In Platt’s [17] RAN algorithm the weight vector update in (11) reduces to

\[ w_k = w_{k-1} + \lambda_k \psi_k e_k, \] (17)

where \( \lambda_k \) is the learning rate which can be a constant or time varying. Meanwhile the new parameter assignments for the new basis function are simplified to (15) without the need for the second order weight covariance matrix \( P_k \) in (16).

2.1. Pruning strategy for sequential learning algorithms

The pruning strategy is based on Yingwei et al. [19] in which the Gaussian kernels which show the least contribution to the model output for the past \( M \) sample instants are eliminated. The pruning procedure can be summarised as follows.

- Compute the output of all the Gaussian kernel functions, \( i = 1, 2, \ldots, m \).

\[ r_{k(i)} = h_i \exp \left( -\frac{\|x_k - e_k\|^2}{\sigma_i^2} \right). \] (18)

- Find the largest absolute Gaussian basis function output value

\[ r_{\text{max}} = \max \left( |r_{k(i)}| \right), \quad i = 1, 2, \ldots, m. \] (19)

- Determine the normalised contribution factor for each basis function

\[ \kappa_i = \frac{|r_{k(i)}|}{r_{\text{max}}}, \quad i = 1, 2, \ldots, m. \] (20)

- If \( \kappa_j < \delta \) for \( M \) consecutive sample instances then prune the \( j \)th hidden neuron and reduce the dimensionality of \( w_k, \psi_k \) and \( P_k \) accordingly.

Yingwei et al. [19] called the combination of this pruning strategy with RAN-EKF a minimal RAN (MRAN).

3. Decomposed training

The EKF covariance matrix, \( P_k \) involves \( O(N_w^2) \) storage requirements and computational complexity, where \( N_w = \text{dim}(w_k) \). As training proceeds, \( P_k \) grows
with the number of network parameters and quickly becomes a limiting factor in the use of EKF for on-line applications. However, by discarding the inter-neuron weight correlations, as proposed by Jankowski and Kadirkamanathan, $P_k$ can be decomposed on a neuron by neuron basis leading to a much sparser matrix. Decomposition of the covariance matrix can be further extended and applied to DRANs where $\hat{P}_k$ (covariance matrix of DRAN) is also partitioned with respect to the direct-link and RBF parameters. This corresponds to an additional layer level decomposition and leads to the following reduced memory implementation.

\[
\hat{P}_k \cong \begin{bmatrix}
P_{DL}^k & 0 \\
0 & \hat{P}_k \end{bmatrix}_{\tilde{N}_u \times \tilde{N}_u},
\]

(21)

\[
\hat{P}_k \cong \begin{bmatrix}
[\hat{P}_{k(1)}]_{n_u \times n_u} & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & \cdots & [\hat{P}_{k(m-1)}]_{n_u \times n_u} & \cdots \\
0 & 0 & \cdots & [\hat{P}_{k(m-1)}]_{n_u \times n_u}
\end{bmatrix}_{\tilde{N}_u \times \tilde{N}_u}.
\]

(22)

Here $\hat{P}_{k(i)}$ represents the correlation matrix (or covariance matrix) for the $i$th neuron weights and $P_{DL}^k$ is represents the covariance matrix of the direct link weights. The numerical complexity and memory requirements for the decomposed covariance matrix are $O(m \times n_u^2)$ compared to $O(m^2 \times n_u^2)$ for $P_k$ in conventional full matrix second-order sequential learning. A graphical interpretation of the sparseness achieved is shown in Fig. 1.

The gradient vector can be equivalently decomposed as

\[
\psi_k = \begin{bmatrix}
\psi_{k(1)}^T \\
\psi_{k(2)}^T \\
\vdots \\
\psi_{k(m)}^T
\end{bmatrix}^T.
\]

(23)

Fig. 1. Graphical interpretation of layer-level and neuron-level decomposition: (a) Layer-level decomposition; (b) Neuron-level decomposition.
There is also the potential for parallel implementation [15] as the weight vector update is decomposed into \( m \) sub-algorithms:

\[
\begin{align*}
\mathbf{w}_{k+1}(i) &= \mathbf{w}_k(i) + \mathbf{\hat{P}}_k(i)\psi_k(i)\mathbf{e}_k, \\
\mathbf{w}_k(i) &= [c_i^T \sigma_i h_k]^T, \\
\psi_k(i) &= \left[ \frac{\partial y_k}{\partial \mathbf{w}_k(i)} \right]_{n \times 1}, \\
\mathbf{\hat{P}}_k(i) &= \mathbf{\hat{P}}_{k-1}(i) - \frac{\mathbf{\hat{P}}_{k-1}(i)\psi_k(i)^T\mathbf{\hat{P}}_{k-1}(i)}{1 + \psi_k(i)^T\mathbf{\hat{P}}_k(i)\psi_k(i)}, \quad i = 1, 2, \ldots, m
\end{align*}
\]

The direct link weights are updated based on the following procedure

\[
\begin{align*}
\mathbf{b}_k &= \mathbf{b}_{k-1} + \mathbf{\rho}^{\text{DL}} x_k e_k, \\
\mathbf{\rho}^{\text{DL}} &= \frac{\mathbf{\rho}^{\text{DL}} \mathbf{x}_k^T \mathbf{x}_k - \mathbf{\rho}^{\text{DL}}}{1 + \mathbf{x}_k^T \mathbf{\hat{P}}_{k-1} \mathbf{x}_k},
\end{align*}
\]

The resulting topology–algorithm combination, a direct-link resource-allocating network with decomposed EKF weight adaptation, will be referred to as DRAN-DEKF.

### 3.1. Decomposed RLM (DRAN-DRLM)

A major advantage of decomposing \( \mathbf{\hat{P}}_k \) (for RANs or DRANs) is the size of each of the covariance sub-matrices \( \mathbf{\hat{P}}_{k(i)} \) is invariant during training in comparison to \( \mathbf{P}_k \) (for RAN-EKF) which changes dimension from one iteration to the next. This facilitates the application of the powerful decomposed recursive Levenberg–Marquardt training algorithm [3] to sequential learning of DRBFs (DRAN-DRLM). The main difference between DRLM and DEKF is that in DRLM a regularisation term \( \rho \) is added to one of the diagonal element of \( \mathbf{\hat{P}}_{k(i)} \) (of DEKF) at each sample instant. The routine for updating each \( \mathbf{\hat{P}}_{k(i)} \) using DRLM is given as follows:

\[
\begin{align*}
\mathbf{S}_{k(i)} &= \Lambda + \mathbf{\Omega}_{k(i)}\mathbf{\hat{P}}_{k-1(i)}\mathbf{\Omega}_{k(i)}^T, \\
\mathbf{\hat{P}}_{k(i)} &= \mathbf{\hat{P}}_{k-1(i)} - \frac{\mathbf{\hat{P}}_{k-1(i)}\mathbf{\Omega}_{k(i)}\mathbf{\Omega}_{k(i)}^T\mathbf{\hat{P}}_{k-1(i)}}{1 + \mathbf{\psi}_{k(i)}^T \mathbf{\hat{P}}_{k(i)} \mathbf{\psi}_{k(i)}}, \\
\mathbf{\Omega}_{k(i)}^T &= \begin{bmatrix} 0 & \ldots & \mathbf{\psi}_{k(i)}^T & \ldots & 0 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix}^{-1} \quad \text{and} \quad i = 1, \ldots, m,
\end{align*}
\]

where \( \mathbf{\Omega}_{k(i)} \) is a \( n_w \times 2 \) matrix whose second column has only one non zero element located at \( k_{r(i)} \mod n_w + 1 \). The scalar \( k_{r(i)} \) is a counter that keeps track of the age of the decomposed kernel function \( \phi \) and is used to determine the location of the non-zero element in \( \mathbf{\Omega}_{k(i)} \) at each iteration. It is initialised to 1 when the \( i \)th basis function is created and incremented every weight update thereafter. If the \( i \)th basis function is
being removed, during pruning, the scalar $k_i$ will be discarded automatically. A global optimisation of the linear weights of the direct-link network is implemented as in and (27) and (28).

4. Minimal update algorithms

Minimal weight update algorithms are motivated by memory and time constraints, particularly in real-time modelling and control applications. In early work on RANs with minimal update by Bomberger and Seborg [5] the weight update rule was restricted to the linear connections of RBF neurons which had significant influence over the model output at a given time. Junge and Unbehauen [9] and Li et al. [11] proposed limiting weight updates to the parameters of a single winner neuron to further improve computational efficiency. Here the minimal update winner neuron concept is extended to DRAN-DEKF and DRAN-DRLM training as follows.

4.1. Minimal decomposed algorithm

Rather than adapting all the decomposed network parameters as in (25)–(26), the minimal update DRAN-DEKF algorithm (DRAN-mDEKF) updates only the weights of the neuron whose centre is closest to the current input vector $x_k$. Thus:

$$c = \arg\min_i \|x_k - c_i\|, \quad i = 1, 2, \ldots, m$$

$$w_{k(c)} = w_{k-1(c)} + \hat{P}_{k(c)} \psi_{k(c)} c_k,$$

$$w_{k(c)} = \begin{bmatrix} e^T_c & \sigma_c & h_c \end{bmatrix}^T, \quad \psi_{k(c)} = \left[ \frac{\partial y_k}{\partial w_{k(c)}} \right]_{n_w \times 1},$$

$$\hat{P}_{k(c)} = \hat{P}_{k-1(c)} - \frac{\hat{P}_{k-1(c)} \psi_{k(c)} \theta_{k(c)}^T}{1 + \psi_{k(c)}^T \hat{P}_{k(c)} \psi_{k(c)}} \hat{P}_{k-1(c)}.$$

Alternatively, covariance matrix $\hat{P}_{k(c)}$ can be estimated using DRLM and updated as follows:

$$S_{k(c)} = \Lambda + \Omega_{k(c)}^T \hat{P}_{k-1(c)} \Omega_{k(c)},$$

$$\hat{P}_{k(c)} = \hat{P}_{k-1(c)} - \hat{P}_{k-1(c)} \Omega_{k(c)} \Sigma_{k(c)}^{-1} \Omega_{k(c)}^T \hat{P}_{k-1(c)},$$

$$\Omega_{k(c)}^T = \begin{bmatrix} \psi_{k(c)}^T & 0 \ldots & 0 \end{bmatrix} \quad \text{and} \quad \Lambda = \begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix}^{-1},$$

where $\Omega_{k(c)}$ is a $n_w \times 2$ matrix whose second column has only one non-zero element located at $k_i \mod n_w + 1$. Counter $k_i$ is incremented every time $w_{k(c)}$ is updated. The direct link network weights are unaffected by this modification and are updated...
using (27) and (28) as before. The resulting algorithm is referred to as minimal DRAN-DRLM (DRAN-mDRLM).

5. Simulation results

5.1. Benchmark problems

Three benchmark modelling problems are used in the evaluation of the proposed DRBF sequential learning algorithms.

Time series prediction benchmark: the objective here is to model a simulated nonlinear time series described by the nonlinear difference equation

\[
y_k = [0.8 - 0.5 \exp(-y_{k-1}^2)]y_{k-1} - [0.3 - 0.9 \exp(-y_{k-1}^2)]y_{k-1}^2 + 0.1 \sin(\pi y_{k-1}).
\]

(39)

The neural model is based on the nonlinear autoregressive (NAR) model structure, that is

\[
y_k = f(y_{k-1}, y_{k-2}) + \eta_k,
\]

(40)

where the noise \(\eta_k\) is a Gaussian white noise sequence with zero mean and variance 0.01.

Coupled-tank system benchmark: modelling the nonlinear dynamics of a two tank system [7] (shown in Fig. 2) is used as the second benchmark. The two tanks are connected by a short pipe and the amount of liquid flowing into the tanks is regulated by a control valve. The identification problem considered here is the prediction of liquid level \(h_2\) of the second tank (Tank 2) given the input flow \(q_1\) to the first tank (Tank 1). The network input vector for the DRBF model is

\[
x_k = \begin{bmatrix} h_2(k-1) \\ q_1(k-1) \end{bmatrix}
\]

(41)

and the output is \(h_2(k)\). Complete details of the benchmark can be found in [7].

Continuous stirred tank reactor benchmark: Continuous stirred tank reactors (CSTR) are highly nonlinear systems and as such useful benchmarks for neural network modelling techniques. The CSTR considered here is a single-input–single-output (SISO) one, where the output is the product concentration, \(C\) and the input is the coolant flow rate, \(q\). The chemical reaction which produces the compound inside the reactor is an exothermic process, which raises the temperature, and reduces the reaction rate. The objective is to control the measured concentration of the product, \(C\), by manipulating the coolant flow rate, \(q\). The CSTR equations are given by

\[
\frac{dT}{dt} = \frac{q_d}{V}(T_d - T) + K_1 C \exp\left(-\frac{E}{R \cdot T}\right) + K_2 q \cdot \left[1 - \exp\left(-\frac{K_3}{q}\right)\right] \cdot (T_{\text{cf}} - T),
\]

(42)
\[
\frac{dC}{dt} = \frac{q_i}{V} (C_i - C) - K_o C \exp \left( \frac{-E}{R \cdot T} \right). \tag{43}
\]

Details of the equation constants, nominal setting and operating conditions for the CSTR benchmark can be found in [13]. Here the process is identified using a neural model assuming a second order nonlinear autoregressive with external input (NARX) model structure. The network input vector is thus of the form
\[
x_k = \begin{bmatrix} C_{k-1} & C_{k-2} & q_{k-1} & q_{k-2} \end{bmatrix}
\tag{44}
\]
and the output is the product concentration at the \( k \)th sample instant, \( C_k \).

5.2. Performance evaluation

To get an indication of the global modelling quality of a DRAN at a particular iteration of the sequential learning process model performance is measured in terms of the percentage normalised mean squared error (%NMSE),
\[
J(w_k) = \frac{100}{p} \sum_{n=1}^{N_v} (y(x_n, w_k) - d_n)^2 / \sum_{n=1}^{N_v} d_n^2 \tag{45}
\]
computed over an \( N_v \) pattern representative test data set, \((x_n, d_n)\). Figs. 3(a) and 4(a) show the typical evolution of this performance index for the coupled-tank and CSTR benchmark problems with each of the four sequential learning algorithms under consideration. The learning curves for RAN-EKF are also included for comparison. Fig. 3(b) also shows the typical growth of a DRAN during training while Fig. 4(b) demonstrates the parallel modelling performance achievable with a minimal update-DRLM trained DRAN for the CSTR benchmark.
Tables 1–3 summarise the performance of each of the sequential learning algorithms discussed in the paper (with and without pruning) on the three benchmarks following 1000 iterations of training. For comparison purposes results are also included for RAN-EKF and DRAN-EKF. The recorded performance indices are the mean NMSE \( E[\|J(w_k)\|] \) computed over the last 500 iterations, the final number of hidden neurons \( m \) and the corresponding number of network parameters \( N_w \). The user defined algorithm parameters were the same for each problem and are as given in Table 4. Finally, Table 5 summarises the approximate memory and computational requirements of each algorithm.

These results show that, in addition to providing significant savings in memory and computational requirements, the decomposed EKF and RLM algorithms consistently yield better NMSE performance than DRAN-EKF. The results also show that the minimal update implementations are promising alternatives to DRLM and DEKF as they have an even lower computational cost yet achieve comparable performance. When used in conjunction with pruning the proposed algorithms maintain their superiority while at the same time producing parsimonious networks.

The decomposed algorithms significantly outperform conventional RAN-EKF and MRAN, but a comparison with the results for DRAN-EKF reveals that this is principally due to the benefits offered by the DRAN architecture.

An examination of the NMSE performance of the different EKF and RLM implementations shows that RLM is marginally superior to EKF. This is consistent with other comparisons of the algorithms reported in the literature [3,14].

The superior NMSE achieved by the decomposed training algorithms may seem remarkable given that they discard a significant proportion of the curvature information in computing the weight updates. However, this is in fact the reason for their success. The stochastic nature of the covariance matrix as it is computed recursively over several iterations and the fact that it varies as a function of RAN
Fig. 4. Sequential learning performance on CSTR benchmark (without pruning). (a) %NMSE learning curve for CSTR benchmark; (b) Parallel model obtain using DRAN-mDRLM.
### Table 1
Performance of sequential learning algorithms on the time series benchmark

<table>
<thead>
<tr>
<th>Without pruning</th>
<th>With pruning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\text{NMSE})$</td>
<td>$m$</td>
</tr>
<tr>
<td>$E(\text{NMSE})$</td>
<td>$m$</td>
</tr>
<tr>
<td>RAN-EKF(^a)</td>
<td>0.0300</td>
</tr>
<tr>
<td>DRAN-DEKF</td>
<td>0.0320</td>
</tr>
<tr>
<td>DRAN-DRLM</td>
<td>0.0191</td>
</tr>
<tr>
<td>DRAN-mDEKF</td>
<td>0.0223</td>
</tr>
<tr>
<td>DRAN-mDRLM</td>
<td>0.0102</td>
</tr>
<tr>
<td>DRAN-EKF</td>
<td>0.0293</td>
</tr>
</tbody>
</table>

\(^a\)RAN-EKF with pruning is usually referred to as MRAN.

### Table 2
Performance of sequential learning algorithms on the two tank benchmark problem

<table>
<thead>
<tr>
<th>Without pruning</th>
<th>With pruning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\text{NMSE})$</td>
<td>$m$</td>
</tr>
<tr>
<td>$E(\text{NMSE})$</td>
<td>$m$</td>
</tr>
<tr>
<td>RAN-EKF(^a)</td>
<td>0.0628</td>
</tr>
<tr>
<td>DRAN-DEKF</td>
<td>0.0099</td>
</tr>
<tr>
<td>DRAN-DRLM</td>
<td>0.0096</td>
</tr>
<tr>
<td>DRAN-mDEKF</td>
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<tr>
<td>DRAN-mDRLM</td>
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<td>DRAN-EKF</td>
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</tbody>
</table>

\(^a\)RAN-EKF with pruning is usually referred to as MRAN.

### Table 3
Performance of sequential learning algorithms on the CSTR benchmark

<table>
<thead>
<tr>
<th>Without pruning</th>
<th>With pruning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\text{NMSE})$</td>
<td>$m$</td>
</tr>
<tr>
<td>$E(\text{NMSE})$</td>
<td>$m$</td>
</tr>
<tr>
<td>RAN-EKF(^a)</td>
<td>0.2854</td>
</tr>
<tr>
<td>DRAN-DEKF</td>
<td>0.0069</td>
</tr>
<tr>
<td>DRAN-DRLM</td>
<td>0.0042</td>
</tr>
<tr>
<td>DRAN-mDEKF</td>
<td>0.0057</td>
</tr>
<tr>
<td>DRAN-mDRLM</td>
<td>0.0062</td>
</tr>
<tr>
<td>DRAN-EKF</td>
<td>0.0090</td>
</tr>
</tbody>
</table>

\(^a\)RAN-EKF with pruning is usually referred to as MRAN.

### Table 4
User defined parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth Criteria parameters</td>
<td>$\varepsilon_{\text{min}} = 0.3$, $\varepsilon_{\text{max}} = 0.5$, $\gamma = 0.98$</td>
</tr>
<tr>
<td>Pruning method parameters</td>
<td>$M = 30$, $\delta = 0.0001$</td>
</tr>
<tr>
<td>Others</td>
<td>$\rho = 1$, $\lambda = 1$</td>
</tr>
</tbody>
</table>
parameters makes it ill-conditioned and unreliable. Focusing on the near diagonal curvature information is thus beneficial in this respect. Furthermore, decomposed training can be viewed as an approximation to separable recursive least squares which in turn is an approximation to off-line separable nonlinear least squares training. One of the properties of separable nonlinear least squares is that the resulting covariance matrix is better conditioned than the corresponding matrix in conventional nonlinear least squares [18]. Empirical studies have shown that this also holds true for separable recursive least squares [4]. Thus, it is likely that the decomposed and minimal update algorithms also benefit from being better conditioned.

6. Conclusions

Two computationally efficient learning algorithms have been developed for DRANs, a generalisation of RANs where the RBF topology is modified to include direct links between the inputs and the output node. The first is obtained by partitioning the network in terms of the direct-link and RBF parameters and decomposing the RBF parameter covariance matrix on a neuron by neuron basis. The second is a variant of the first in which only the parameters of a winner neuron are updated at each iteration to reduce the computational burden. Results presented for implementations, with and without pruning, show that the decomposed sequential learning approaches consistently outperform DRAN-EKF in terms of the final percentage NMSE achieved. In addition, the algorithms use less memory, require less computation time per iteration and produce more parsimonious networks.

The minimal update algorithms (DRAN-mDEKF and DRAN-mDRLM) are especially noteworthy as they achieve a substantial reduction in computational load compared to DRAN-mDEKF and DRAN-mDRLM without sacrificing NSME performance.
While originally motivated by the search for computationally efficient algorithms, the superior learning capabilities of DRLM and mDRLM make them the first choice algorithms for training DRANs even when memory requirements and real-time constraints are not an issue.

References

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George Irwin is a Professor of Control and Director of the Virtual Engineering Centre at Queen’s University, Belfast, UK. His current research interests include identification and control using neural networks, fuzzy neural systems, local model networks, and multivariate statistics. Much of his work involves industrial collaboration and he was a Director of Anex6 Ltd., a spin out company from his group specialising in process monitoring. He has published over 300 research papers and 6 edited books. Prizes include four IEE Premiums, a Best Paper award from the Czech Academy of Sciences and the 2002 Honeywell International Medal from the UK Institute of Measurement and Control. He is a former Editor-in-Chief of the IFAC journal Control Engineering Practice and current chair of the UK Automatic Control Council.

Prof. Irwin has been elected Fellow of the Royal Academy of Engineering and Member of the Royal Irish Academy. He is a chartered engineer, an IEEE fellow, a fellow of the IEE and a fellow of the Institute of Measurement and Control.