Developing Mathematical Knowledge for Teaching (MKT) for pre-service teachers: a study of students' developing thinking in relation to the teaching of mathematics

Brien Nolan[a], Majella Dempsey[b], James Lovatt[a] and Ann O'Shea[b]
[a]CASTeL, Dublin City University; [b]Maynooth University

The concept of Mathematical Knowledge for Teaching (MKT) was introduced by Ball and colleagues (Ball, Thames & Phelps, 2008), building on Shulman's (1986) notion of Pedagogical Content Knowledge. MKT is 'the mathematical knowledge needed to carry out the work of teaching mathematics'. In this project, a team of researchers at two Irish universities studied the development of MKT in two groups of pre-service teachers. The project aimed to help students develop their own MKT, and to develop a richer conception of the role of mathematics content knowledge in teaching, through a series of workshops designed and delivered by the authors. The students' awareness and level of MKT was investigated using pre- and post-intervention questionnaires. We describe the intervention and present the findings from the analysis of the data collected. In particular, we describe how the group's view of the mathematical work of a teacher changed over the course of the project.

Keywords: teacher education; Mathematical Knowledge for Teaching

Introduction

The study of content knowledge appropriate to teaching was reinvigorated by Shulman's introduction of the concept of Pedagogical Content Knowledge (PCK) (Shulman, 1986). Shulman noted the existence of "a blind spot with respect to content that now characterizes most research on teaching" (Shulman, 1986, pp.7-8). In the case of mathematics, this concept was refined and extended by the introduction of the idea of Mathematical Knowledge for Teaching (MKT), defined as "the mathematical knowledge needed to carry out the work of teaching mathematics" (Ball, Thames & Phelps, 2008, p.395). The MKT framework serves different purposes and has allowed for the identification of those areas of a teachers' mathematical knowledge that support student achievement (Hill, Rowan & Ball, 2005; see also Ma 1999). Thus MKT has had an influence on mathematics teacher education (Ball, Sleep, Boerst & Bass, 2009; Graeber & Tirosh, 2008). Two strands of research associated with MKT are of direct relevance here: investigating how MKT can be measured (Hill, Ball & Schilling, 2004), and how the level of MKT held by teachers can be raised (Suzuka et al. 2009; Bell, Wilson & Higgins, 2010; Steele, Hillen & Smith, 2013).

In this paper, we report on an ongoing research project that deals (principally) with these two issues: assessing pre-service teachers' level of MKT, and finding effective ways of building these teachers' MKT in the Irish second-level context. We also report on the development of the teachers' awareness of MKT – the degree to which these pre-service teachers are aware of the variety of teaching tasks that require mathematical knowledge. The study is based on two cohorts of pre-service teachers undertaking concurrent mathematics teacher education programmes in Dublin City University (DCU; N=17) and Maynooth University (MU; N=13). We will refer to the subjects as '(the) teachers'. Both cohorts were in the second year of their four-year programmes. The students undertook their first period of school-based work during the academic semester in which this project took place. However, this teaching placement was not formally linked to the present project. Thus (for example) the assessment tasks of this project were not linked directly to the teachers' classroom work. The different sets of quantitative and qualitative data gathered during the course of the study provide us with a picture of the two groups' evolving views of the mathematical work of teaching mathematics. We will consider this with the aid of Goodwin's concept of Professional Vision (Goodwin, 1994).

Mathematical Knowledge for Teaching

The MKT framework deals with two key domains of teacher knowledge discussed by Shulman (1986): PCK and Subject Matter Knowledge (SMK). The empirically devised framework, based on observations of US third grade mathematics classes, describes different categories of these two domains.

The SMK domain is comprised of three sub-domains: Common Content Knowledge (CCK), Horizon Content Knowledge (HCK) and Specialized Content Knowledge (SCK). CCK refers to mathematical knowledge (and skills) not unique to teaching. The link with teaching comes from the need for the teacher to be able to do the mathematical work assigned to students. HCK comprises knowledge of how different topics are related over the span of mathematics included in (and just beyond) the curriculum. This knowledge is important for the appropriate sequencing of taught content. SCK refers to mathematical knowledge and skills unique to teaching. This includes the ability to carry out such tasks as looking for patterns in student errors and determining if nonstandard approaches are valid and generalizable. This requires unique mathematical understanding and reasoning (see Ball et al., 2008).

Ball et al. (2008) have also devised a decomposition of Shulman's PCK domain. In the MKT framework, this domain comprises Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS) and Knowledge of Content and Curriculum (KCC). KCT combines knowing about teaching and knowing about mathematics. This comes into play in various ways, but perhaps most importantly when mathematical knowledge and choices in relation to instructional options and purposes come together. KCT is also likely to be involved in contingent teaching actions, where, for example, a teacher decides which student contributions to pursue and which to put on hold. KCS involves knowledge that combines knowing about students and knowing about mathematics in a way that enables teachers to (for example) anticipate what students may think and what they will find confusing, interesting and motivating, and to interpret students' (not fully coherent) spoken words and written work. The concept of KCC will not be discussed here.

As noted above, MKT has been found useful in a variety of ways. It provides a framework for the discussion of teachers' mathematical knowledge, and informs the development of teacher education programmes and the design of support materials for teachers. Research showing that teachers' level of MKT correlates positively with student achievement demonstrates the importance of the concept (Hill, Rowan & Ball, 2005). Thus it is important to ask: how can MKT be developed and how can MKT be measured? In relation to the latter, the work of the Learning Mathematics for Teaching project team is of particular importance (Hill, Ball & Schilling, 2004; LMT, 2008). In this work, test items that assess MKT were designed and validated. Sample items have been released (LMT, 2008), and these were used in the present project.

Each item is aligned with a particular sub-domain of MKT. Overall, the released items do not by themselves provide a measure of an individual's or group's level of MKT (LMT, 2008): we will explain their role in this project below.

In relation to how MKT may be developed, a variety of approaches have been taken (Suzuka et al. 2009; Bell et al., 2010; Steele et al., 2013). We describe below the approach used in the present project.

Assessing and building MKT

The project had two main elements. The first (the assessment element) focussed on assessing MKT, both from the perspective of the level of MKT held by the teachers, and their awareness of MKT. The latter describes the degree to which the teachers are aware of the variety of mathematical work involved in teaching mathematics. In the second element (the teaching element), we designed and delivered components of a mathematics pedagogy course which had the purpose of supporting the students' development of their MKT.

We will see that the assessment of both level and awareness of MKT arise in the teaching element of the project, but we summarise here the assessment element in which the primary focus was on assessing MKT. As noted, we consider this in two different ways: the teachers' level of MKT, and the teachers' awareness of MKT. For both, a pre- and post-test approach was used. Regarding the assessment of level of MKT, a subset of the LMT released items was used. We emphasize that we do not interpret the teachers' performance on these items as providing a measure, on its own, of their level of MKT, in line with the recommendations of (LMT, 2008). We view the results (discussed below) as comprising just one part of a broader picture. The LMT released items deal with mathematics content encountered in the Irish context in the upper-primary and lower-secondary years. In particular, many of the items are immediately relevant to the Common Introductory Course, which is indicated for the teaching of mathematics in the first year of secondary school (NCCA, 2012). Thirteen items were selected, dealing mainly with arithmetic of integers and fractions, as well as basic geometry. Of these, five were categorised (by their authors) as assessing Common Content Knowledge, five as assessing Knowledge of Content and Students with the remaining three focussing on Knowledge of Content and Teaching.

The assessment of the teachers' awareness of MKT was carried out through the use of a qualitative survey comprising these two open-response questions:

> What *specific* knowledge and skills do you think a mathematics teacher needs?
>
> List different teaching situations where a teacher uses his or her knowledge of mathematics. (It may help you to think in terms of different tasks that a teacher carries out over the course of a day, a week, a term, a school year…)

These questions were aimed at determining the extent to which the teachers could report on the variety of mathematical work carried out by mathematics teachers. With the different emphases of the two questions, we intended to elicit responses which would encapsulate, on the one hand, notions of inherent attributes held by mathematics teachers (knowledge and skills), and the enactment of those attributes (teaching situations) on the other. We note that the problem of seeing MKT that may be held by a teacher enacted in a teaching context has been highlighted in Suzuka et al., (2009) and Stylianides & Stylianides, (2014): it is particularly pertinent here given that we were observing the teachers' MKT outside the mathematics class context.

The teaching element of the project took place during the first semester of the academic year 2013-14 in DCU and over two semesters in MU. Following the delivery of the level and awareness surveys in our first sessions with the two separate cohorts, we engaged the teachers in a variety of activities. The teachers undertook readings and guided discussion of articles on MKT: learning what it is and why it is relevant. We presented a mini-course on the mathematics of fractions, focussing on the ways in which second level teachers need to know this branch of mathematics (Wu, 2011). This engaged the teachers in activities drawn from and similar to the LMT released items document (LMT, 2008), and similar activities that focus on student work on fractions. The teachers analysed a video recording of a mathematics class, and were asked to identify instances (and omissions) of the application of MKT. The final part of the teaching element involved the discussion of the development of rich tasks for learners (Breen & O'Shea, 2010) by using MKT and a Levels of Cognitive Demand framework (Smith & Stein, 1998). Data were gathered related to these activities in the form of different assessment items undertaken by the teachers. These were exercises involving the application of KCS and KCT in teaching fractions, a video analysis exercise rating the application of MKT, and an exercise that required the teachers to explicitly highlight their use of MKT in the design of rich mathematical tasks for learners.

Teachers' level of MKT

We focus on the data generated and analysed in the assessment element of the project. We will report on the qualitative data generated in the teaching element elsewhere.

The 13 LMT test items used gave rise to 18 individual questions. Each individual question was marked as either correct or incorrect. A total of N=14 students undertook both the pre- and post-test survey, with the same bank of questions being used in both. The pre-test survey took place in late September 2013, and the post-test in April 2014. The mean (respectively median) score on the test increased from 7.3 (respectively 7) on the pre-test to 9.6 (respectively 9) on the post-test. These differences are statistically significant ($p=0.05$; related samples Wilcoxon signed rank test). There were 55 gains from pre-test to post-test (i.e. 55 questions that were answered incorrectly in the pre-test but correctly in the post-test) and 22 losses. Thus we see some evidence of an increase in the teachers' level of MKT.

Focussing on individual test items gave rise to some interesting observations. For example, a test item related to KCT asked teachers to identify (from three choices) the best sequence of leading questions that would help a student with the question "How many 4s are there in 3?" The teachers' pre- and post-test responses showed an overall migration from a choice indicative of a rote-learning approach (option c) to a choice that engenders an understanding of the relevant number facts (option b). In the pre-test, 10 of 14 teachers chose option c and a single teacher chose option b; in the post-test these figures changed to 6 and 6 respectively. We see this migration as being underpinned by an improved understanding of the concept of fractions, and of a changing belief that students should be taught for understanding.

Teachers' awareness of MKT

The data from the awareness survey were analysed using a grounded theory approach (general inductive analysis) described by Thomas (2006). Two researchers separately coded the data: 13 categories emerged following (i) analysis of the pre-test data; (ii) review; (iii) analysis of the post-test data. The MU data were then categorised by two researchers, with 78% inter-rater reliability on initial categorisation.

In Tables 1 and 2, we list the categories which occurred most frequently in the pre- and post-test surveys, indicating the number of occurrences within those categories for both surveys. We give descriptions of two of these categories:

> *Common teaching knowledge, skills and attributes:* this category refers to features of teaching that are almost entirely independent of content, and in particular do not require the use of any mathematical knowledge. It includes such things as patience and the ability to impose discipline.
>
> *Knowledge of students:* this refers to situations in which the teacher demonstrates or needs to have knowledge of the students' mathematical abilities. This includes references to needing to know what topics or approaches students find difficult.

Category name	Pre-test frequency	Post-test frequency
Common teaching knowledge, skills and attributes	26	13
Content knowledge	17	14
Explaining and instructing	13	11
Knowledge of students	9	17

Table 1: Highest frequency categories in the pre-test survey.

Category name	Pre-test frequency	Post-test frequency
Listing MKT categories	0	22
Evaluation of learning	7	19
Knowledge of students	9	17
Planning and sequencing	9	17

Table 2: Highest frequency categories in the post-test survey.

In interpreting these categories, we have found it useful to ask how they relate to MKT. We consider that the different categories indicate the awareness on the part of students of the variety of situations in which the teacher uses mathematical knowledge. However the question of which aspect of MKT may be in play cannot, in general, be rated. The emergent categories can also be read using the lens of professional vision (Goodwin, 1994). This emphasizes two key undertakings on the part of novices engaged in a complex social activity (teaching in this case). These are 'highlighting' – what aspects of teaching do the teachers consider to be important? – and 'coding': what rationale do they give for this judgement?

Seen through this lens, we note a changing view of the role of the teacher, with a greater focus on the learners' needs and their agency in the mathematics classroom. This is evidenced in, for example, the increase in the number of responses in the 'Evaluation of learning' and 'Knowledge of students' categories. We also see evidence that the teachers highlight (in Goodwin's sense) MKT: there is a marked decline in the 'Common teaching…' category, accompanied by an increase in the number of responses that explicitly mention mathematical work.

The high frequency of responses in the 'Listing MKT categories' needs to be interpreted carefully. Here, responses comprised simply writing down the name or acronym of an MKT sub-domain (e.g. KCS). This arose only in the post-test data from DCU in responses to the first question. Contrary to some of the other data, this seems to be indicative of a superficial engagement with the concept of MKT.

Conclusions

The data we have analysed thus far provides tentative evidence of growth in the teachers' level of MKT, and a richer view of mathematics teaching that focusses more on the learners' role and needs. We also see MKT and related concepts being incorporated in the teachers' professional vision. We expect that we will gain more reliable insights into these initial findings when the assessment items of the teaching element of the project are fully analysed. The concepts of MKT and professional vision will lie at the heart of this analysis.

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