Using Bidimensional Regression to analyse Cognitive Maps

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Abstract

Bidimensional regression is an analytical technique that allows the comparison of two sets of paired coordinate points, determining whether they are spatially correlated. It is a useful technique in cognitive mapping because it can be used to measure the association between where a subject thinks a place is and where that place is in reality. As such it determines the degree of configurational knowledge (knowledge of the relative positions of places) a subject possesses, as displayed in the spatial products (external representations of their everyday geographic knowledge, such as sketch maps) they provide in experiments.

Introduction

Spatial cognition is defined by Hart & Moore (1973, p.248) as:

"...the knowledge and internal or cognitive representation of the structure, entities, and relations of space; in other words, the internalized reflection and reconstruction of space in thought."

Environmental cognition on the other hand refers to:

"the awareness, impressions, information, images, and beliefs that people have about environments...it implies not only that individuals and groups have information and images about the existence of these environments and of their constituent elements, but also that they have impressions about their character, function, dynamics, and structural interconnectedness, and that they imbue them with meaning, significance, and mythical-symbolic properties" (Moore and Golledge, 1976, p.xii).

Cognitive mapping is the marriage between spatial and environmental cognition, and is essentially 'place cognition' as described by Hart and Conn (1991). Rather than dealing exclusively with either the spatial aspect or the environmental aspect of how we think about everyday environmental and geographic data, cognitive mapping combines the relevant sections of the two. Downs and Stea (1973, p.7) state that:
"Cognitive mapping is a process composed of a series of psychological transformations by which an individual acquires, stores, recalls, and decodes information about the relative locations and attributes of the phenomena in his everyday spatial environment."

In a later work (1977, p.6) they further their definition by explaining that:

"Cognitive mapping is a process of doing; it is an activity that we engage in rather than an object we have. It is a way in which we come to grips with and comprehend the world around us."

At its most general, a cognitive map is a hypothetical construct which we use to know the environment (Kaplan, 1973a). This construct is thought to exist because it is assumed that people store information about their environment which they then use to make spatial decisions which guide behaviour, and is, in effect responsible for geographic 'survival' knowledge (Kaplan, 1973b; Stea, 1969). This knowledge includes not only the observable physical environment, but also memories of environments experienced in the past and the many social, cultural, political, economic and other environments which impinge both on those past memories and current experience (Golledge and Stimson, 1987).

As such, cognitive maps are not just a set of spatial mental structures denoting relative position, they contain attributive values and meanings. As Wood and Beck (1989, p.25) explain, the cognitive map 'is not independent of meaning, of role, of function, of need, of end, and of purpose'. This distinction leads to the conclusion that a cognitive map includes knowledge about places as well as knowledge about the spatial relationships between places (Kaplan, 1976) and that cognitive maps involve the integration of 'images, information and attitudes about an environment' (Spencer and Blades, 1986, p.240). They are 'representations of objects and their associations' involving generic and motivational information (Kaplan, 1973b).

Cognitive maps then, are a series of knowledge structures which consist of different levels of detail and integration (Golledge and Timmermans, 1990). These knowledge structures develop with age and education, and by combining different knowledge structures and information using cognitive processes relating to perception, storage, retrieval and reorganization that interact with memory structures, a cognitive map is formed for specific tasks (Golledge et al., 1985). This, if interpreted literally, means there is no one cognitive map in memory but rather we construct them for specific events (Siegel, 1982; Book, 1989). In this respect cognitive maps are dynamic.

It must also be recognised that cognitive maps are not independent of time and space and that 'since each environment exists in a time-space context, so too will cognitions of those environments' (Moore and Golledge, 1976, p.11). In summary then the cognitive maps constructed from the knowledge store contain information concerning spatial relations and environmental attributive data within a space-time context, allowing the possessor to operate within an environment. They are 'complex, highly selective, abstract and generalised' structures which are 'incomplete, distorted, schematised, and augmented' (Downs and Stea, 1973, p.18). After all they are a 'product of our experience, not of precise measurement' (Kaplan, 1973a, p.276).

**Configurational Knowledge**

Researchers normally distinguish three main types of cognitive map knowledge. The two basic types are declarative and procedural knowledge. A person's declarative knowledge is a mental database of all the information contained in the long term memory. In the cognitive mapping domain this consists of specific places such as landmarks, lines (e.g. paths, edges and boundaries) and areas (e.g. neighbourhoods, cities, and countries) (Golledge, 1992). Procedural knowledge consists of the rules used to synthesise the declarative knowledge database into information which can be used to facilitate an action. For the purpose of cognitive mapping these rules are essentially wayfinding knowledge which direct movement between places, an example of which would be the transformation of path elements into a navigable route. This transformation though does not include the ability to make inferences about routes never experienced.

The highest level of cognitive map knowledge is called configurational knowledge and it surpasses procedural knowledge by incorporating information such as angles, directions, orientation, location and distance apart of places (Golledge, Gale and Richardson, 1987) so the possessor has knowledge of the associations between, and the relative positions of, places and this forms a comprehensive spatial knowledge system (Golledge, 1992). Such knowledge has been termed 'common sense knowledge' by Kuper (1982) and allows inferences and propositions to be made (Allen, 1985).

There are both practical and theoretical reasons for wanting to determine the amount of configuration knowledge which a subject possesses. Firstly, Golledge and Timmermans (1990) have argued that it is as important to determine the dimensions of psychological spaces as it is to determine the dimensions of
physical space. Cognitive maps are constructs which determine and explain our behaviour in space, and if we can discover the geometry, and hence distortions of cognitive maps, we can replace physical space with a cognitive space to be used in a variety of behaviour models (Golledge, 1969; Cadwallader, 1979). It is hypothesised by Gale (1982) that the information received from bidimensional regression, in conjunction with the results of other measuring techniques could also show us how the cognitive map information is stored within the brain. Information concerning the amount of configurational knowledge will also be useful for determining how well the spatial knowledge of a subject is developed for use in development, wayfinding and spatial decision making models, environmental planning and variety of educational uses based around primary school exercises, cartography, remote sensing and the improvement of Geographical Information System’s (GIS) interfaces.

**Bidimensional Regression**

Bidimensional regression is a technique that can give an indication of the amount of configurational knowledge possessed by a subject by comparing the spatial product with objective reality. Because of its statistical preciseness it is preferable to visual assessment. It must be stressed, though, that bidimensional regression can only measure the ‘accuracy’ (also called absolute error, which is the systematic, non-compensating distortion caused by cognitive processes that translate, rotate or scale locations which result in the displacement of a location cue from its actual location (Gale, 1982; Lloyd, 1989)) of configurational knowledge and not the ‘precision’ (also called relative error, which is the areal dispersal of a group’s estimates of a location (Gale, 1982)).

Tobler (1965; 1976; 1978) has made significant advances in the problem of comparing point patterns far beyond orthogonal transformations (Gatrell, 1983). His bidimensional regression method (1965) measures only the association between configurations (Tobler, 1976) and postulates a regression-like relationship, that is basically an extension of ordinary product moment (Pearsonian) correlation and ordinary least squares regression procedures between two set of coordinates. This regression technique is sensitive to rotations, translations, and changes of scale, and calculates how large these are (Tobler, 1976).

The difference between linear and bidimensional regression is that instead of paired one-dimensional observations of the form \( x, y \), one has paired locations of the form \( x, y \) (coordinates of objective places); \( u, v \) (coordinates of cognitive location of places). From these paired couples a spatial correlation can be calculated. It must be noted that in ordinary correlation, phenomena are associated with either the same or different locations, for example, objective sites of towns, and their cognized position as perceived by one individual (Figure 1). The method therefore requires an a priori pairing of locations (Tobler, 1965).

Bidimensional regression compares both sets of coordinates simultaneously. The cognized map \( \{u_i, v_i\} \) coordinates are translated or shifted (thus centring the map), scaled (enlarged or reduced) and rotated to achieve the statistical best fit with the objective geographical locations \( \{x_j, y_j\} \) (Figure 2). The scaling and rotation can be demonstrated by using the equivalent of the regression line in ordinary linear regression, the regression grid, which is produced using bidimensional regression. This grid graphically demonstrates (Figure 3) the amount of scaling, translation and rotation required to allow the \( \{u_i, v_i\} \) set of points to obtain maximum compatibility with the \( \{x_j, y_j\} \) points. Although reflection may be required to make the cognized coordinates correspond with the objective coordinates it cannot be accomplished using the bidimensional regression technique which is not sophisticated enough to perform such a task (Murphy, 1978; Gatrell, 1983).

The simple ordinary linear regression equation can be stated as:

\[ Y = \alpha + b \cdot X + \varepsilon \]

In the two dimensional situation the parameters \( \alpha \) and \( b \) of the standard regression equation become:

\[
\begin{pmatrix}
\alpha \\
\beta
\end{pmatrix} =
\begin{pmatrix}
a_1 \\
a_2
\end{pmatrix}
\begin{pmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{pmatrix}
\]

This translates to bidimensional regression:

\[
\begin{pmatrix}
u_j \\
v_j
\end{pmatrix} =
\begin{pmatrix}
a_1 \\
a_2
\end{pmatrix} +
\begin{pmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{pmatrix}
\begin{pmatrix}
x_j \\
y_j
\end{pmatrix} +
\begin{pmatrix}
v_j \\
v_j
\end{pmatrix}
\]

The parameters \( a_1 \) and \( a_2 \) are analogous to the intercept term and perform the translation; the scaling and the rotation are accomplished by the matrix of \( b_{ij} \)'s
(analogous to the slope coefficient in ordinary linear regression). A rigid euclidean rotation is maintained by constraining \( b_{12} \) to equal \(-b_{21}\), whilst constraining \( b_{22} = b_{11} \) ensures the scale on both the axes is adjusted by the same amount and thus the regression grid remains equilateral (Figure 3) (Murphy, 1978; Gatrell, 1983).

The output in the overlaid maps (Figure 3) can be interpreted thus: \( a_1 \) is the horizontal translation, with a positive value indicating a west-to-east shift and a negative value indicating an east-to-west shift; \( a_2 \) is the vertical translation, with a positive value indicating a south-to-north shift and a negative value indicating a north-to-south shift; the scale, is an index that measures the scale change needed to produce the best fit with a value less than one indicating that \( u,v \) space was contracted to fit the \( x,y \) space, and a scale value greater than one indicates the \( u,v \) space was expanded; the angle, is the angle the coordinates axes must be rotated to produce the best fit, with a positive value indicating a counterclockwise rotation, and a negative value a clockwise rotation (Lloyd, 1989).

**Figure 1** Bidimensional regression. This shows how one set of points on the cognitive plane are related onto the pairings on the objective plane.

**Figure 2** Map transformations. This demonstrates how the map transformations take place.
Figure 3 The bidimensional regression of the cognized space of a group of 11-12 year old boys, demonstrating the scaling, translation and rotation in comparison to the real world.

Waterman and Gordon's (1984) distortion index and boxes represent a continuation of bidimensional regression and is calculated from its results. The distance between the \(\{u,v\}\) and \(\{x,y\}\) coordinates is the quantity that is minimized by the transformations, and it is argued, the most suitable as a basis of comparison between different cognitive maps. They advocate the calculation of the distortion index (DI). The DI ranges between 0 and 100 percent and is a dimensionless value, the size of which indicates the amount of distortion regardless of the scale of the true or cognitive map. This is useful for comparing cognitive maps of say, the same person but at different scales. It is in effect a standardised measure of relative error (Lloyd, 1989). This can be displayed graphically using a box of distortion that indicates the orientation of maximum and minimum distortion for the whole cognitive map, with the box's size indicating the relative amount of distortion. The distortion box is however, only comparable between cognitive maps of the same objective map.

Studies using bidimensional regression to study configurational knowledge
There have been only a handful of studies utilising bidimensional regression for the analysis of spatial products. These have been predominately carried out by geographers, rather than by researchers interested in cognitive maps from other subjects such as psychology, cognitive science, sociology, anthropology and planning.

Golledge and his associates (Richardson, 1981a,b; Gale, 1982; Golledge and Hubert, 1982; Golledge, Rayner and Rivizzingo, 1982; Golledge, Gale and Richardson, 1987) have used bidimensional regression to assess and compare the spatial knowledge of various subjects to objective reality, and have demonstrated its worth as an analytical technique. In this large study, subjects from Columbus, Ohio, were asked to perform a number of cognitive mapping tasks. One of these required them to make a sequence of paired comparison judgements (assigning scale scores between one and nine on the degree of separation; one representing the closest; nine furthest away) between forty nine highly familiar location cues in that city. These distance ratings were then constructed into a two dimensional space using a nonmetric multidimensional scaling (MDS) program. Each individual's two dimensional multidimensional scaling product was then compared to the objective reality using the commonly employed (Kosslyn, Pick and Fariello, 1974; Magana, Evans and Romney, 1981; Kirasic, 1989) CONGRU program of Olivier (1970), which standardises (makes the same scale) and makes congruent (rotates the configurations until the best fit is obtained) sets of configurations before using the bidimensional algorithm to measure the spatial association.

The results of such a study must be interpreted carefully as the results may not demonstrate the amount of configurational knowledge subjects possess. This is because it was possible for the subjects to assess the distance between two places independently, and not solely in relation to all of the other locations. As such procedural or even declarative knowledge could have been used. The MDS method only allows the measurement of latent (potential, unrealised or unknown) configurational knowledge. The results, though, have been used (Golledge et al., 1983) successfully as a control to demonstrate the low levels of spatial knowledge of mentally retarded subjects, who may need extra education to be able to navigate successfully in the urban environment, and by Richardson (1981b) in analyzing the metrics (geometrical rules) of cognitive maps.

The MDS technique may also be used to compare the configurational knowledge of subjects disaggregated by certain characteristics. School children from West Wirral, Merseyside, were required to locate a set number of designated places (nineteen British towns) in relation to two known locations cues (their school and London) (Kitchin, 1992). This spatial cued response method does require configurational knowledge as all places have to be located relatively and not independently. Bidimensional regression was then used to compare the spatial
products estimated to the objective locations to determine if there were configurational knowledge differences between genders, and different age groups, and to analyze the effect of aggregation by performing the analysis on individual and aggregated data sets. Aggregation took place prior to the bidimensional regression and each aggregated cognized point represents an average position of the groups estimates. The results show that there are minimal differences between the sexes, especially at the individual level, and that configurational knowledge generally liaison increases with age (Figure 4).

Given the coordinates of the objective and the cognitive locations of an aggregated group of 11-12 year old male respondents, we can use bidimensional regression to predict the residuals $u$, $v$ (see Table 1) from the equation:

$$
\begin{pmatrix}
    u \\
    v
\end{pmatrix}
= \begin{pmatrix}
    215.18 & 0.8418 & -0.2722 \\
    25.12  & 0.2722 & 0.8418
\end{pmatrix}
\begin{pmatrix}
    x \\
    y
\end{pmatrix}
$$

where:

$$r^2 = 0.7303 \quad \text{Percent fit} = 53.3393$$

$$\text{Scale} = 0.8847 \quad \text{Angle} = 17.96$$

The resulting transformation is displayed in Figure 3. A graphical representation of the results (Figure 5) allows us to visually interpret the amount of distortion, and the grey shaded area is how the group hypothetically cognizes Britain. The vectors show how far and in what direction the cognized locations need to be moved to fit the objective locations and the residual locations of where the cognized locations were expected to be placed according to the model. As can be seen Britain is cognized as being smaller (note the scale value is less than one in Figure 3) and rotated to the right (the angle value is positive). The relatively high $r^2$ value indicates that the locations of the cognized places are still approximately relative to each other and not just randomly located, although the results suggest that children overestimate the distance to near places and underestimate the distance to far places.

The bidimensional regression results in this example are an indication of the amount of configurational knowledge a subject possesses. The results do not explain the reasons for these distortions and only measure the association between the two configurations. The next step in the investigation of configurational knowledge is to try and determine the reasons for these distortions from planimetry by testing a series of hypotheses. There are many factors that could affect the type and amount of configurational knowledge and these can be divided into ten main categories:

1. Environmental deterministic sources (unalterable) e.g. general physical topography, objective distance.
2. Environmental deterministic sources (alterable) e.g. number of turns or intersections along a route, urban structure.
3. Environmental interaction sources e.g. familiarity, mode of travel, travel time.
4. Social circumstances and interaction sources e.g. Education, Socioeconomic status, media, social/verbal mediation, experience of map use.
5. Perceptual filters, perceptual context and anticipatory schemata e.g. senses, current emotional state, what the subject expects.
6. Characteristics of the mapper (determined) e.g. age, gender.
7. Characteristics of the mapper (undetermined) e.g. inner organismic factors such as beliefs, needs, emotions, personality, metacognition.
8. Cognitive style e.g. how a subject approaches a problem.
9. The form, function, structure and contents of the information in the brain.
10. Memory errors.

Many different combinations of these factors could account for the distortions found using the bidimensional regression technique, and without testing them statistically it is difficult to draw any definitive conclusions about the differences from planimetry found in the example. By using visual interpretation it is possible to make tentative hypotheses for the distortions. Those towns with larger populations seem to be located more accurately than those smaller in size. The reasons for this probably include the respondents greater familiarity as a result of direct experience, media and educational coverage. It is likely that the rotation and the scaling, resulting in Scotland being much reduced in size, and ‘dragged’ south is caused by Scotland being cognized as much smaller than England. A possible reason for this may be that the majority of Scotland’s population is concentrated into the central lowlands area, with subjects unaware of how far the Scottish highlands extend beyond this zone. As a result the locations of the Scottish towns reflected this, with points 13 (Aberdeen) and 18 (John O’Groats) being ‘pulled’ southwards. Further investigation would obviously be required to test these hypotheses.

Figure 5 Map showing the objective, cognitive and residual locations of 11 - 12 year old boys.
Table 1: Cognitive, objective and predicted coordinates

<table>
<thead>
<tr>
<th>Cognitive x and y coordinates</th>
<th>Objective u and v coordinates</th>
<th>Predicted u and v coordinates</th>
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<tr>
<td>445.00 325.00</td>
<td>815.00 395.00</td>
<td>793.71 579.46</td>
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<td>695.00 955.00</td>
<td>875.00 1015.00</td>
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<td>690.00 425.00</td>
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Conclusion
This paper has aimed to highlight the utility of bidimensional regression to cognitive mapping, through its ability to measure the 'accuracy' of the configurational knowledge of subjects spatial products. Such information is important in cognitive mapping because it highlights the distortions involved in cognitive map knowledge. It provides the researcher with information which can then be used in models which try to discover the reasons for such distortions, and in developing and testing theories of spatial behaviour and decision making for use in a number of applications such as educational and environmental planning.

References


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Glaciers and climate: a sensitive balance

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Abstract

The relationship between glaciers and climate represents a sensitive balance. Glaciers can respond to a variety of climatic factors, such as changes in temperature, precipitation and wind direction, and different glaciers may respond in different ways to the same climatic events. By seeking to understand this relationship we can begin to use past fluctuations in the size of glaciers to reconstruct local and regional weather conditions. However, the relationship works both ways, in that glaciers also influence climate. Small glaciers only influence climate locally but major ice sheets, such as those that developed during glaciations, must have had a profound impact on world weather patterns. Although the build-up of ice sheets in response to global cooling is quite easy to envisage, the controls on deglaciation are still poorly understood. In fact, we still do not really understand why, during its early history, the Earth did not freeze over completely.

Introduction

Glaciers exist in equilibrium with climate. When it gets colder they grow and when it gets warmer they shrink. That’s the story I was told at school and even at University. It’s simple, it’s boring and it’s not really true. Glaciers are wonderfully complex things. Like the mythical modern man, they are at once powerful, beautiful and sensitive. It is their sensitivity, with regard to climate, that I wish to discuss.

The concept of equilibrium is useful in understanding how glaciers work. They act rather like a conveyor belt, transporting ice away from areas where snow is accumulating faster than it is melting and into areas where it will melt faster than snow can accumulate. The top part of a glacier will therefore be an area of net accumulation and the lower part an area of net ablation (melting, calving, etc.). Where the two areas meet there is an imaginary ‘equilibrium’ line. If there is an increase in temperature then, ‘all other things being equal’, the rate at which the glacier is melting will increase and the position of the equilibrium line will move up-glacier. The position of the snout will also move upslope, leaving an area of bare ground in front of it. Conversely, if the climate gets colder, ‘all other things being equal’, the glacier is likely to melt less quickly so that the equilibrium line will migrate down-glacier and the snout will advance.

In theory this would suggest that glaciers are very convenient ‘paleo-thermometers’. If we can find out when they advanced and retreated we should...