Journal of Financial Economics 21 (1988) 255–289. North-Holland

RISK AND RETURN IN AN EQUILIBRIUM APT
Application of a New Test Methodology*

Gregory CONNOR

University of California, Berkeley, CA 94720, USA

Robert A. KORAJCZYK

Northwestern University, Evanston, IL 60208, USA

Received June 1986, final version received March 1988

We use an asymptotic principal components technique to estimate the pervasive factors influencing asset returns and to test the restrictions imposed by static and intertemporal equilibrium versions of the arbitrage pricing theory (APT) on a multivariate regression model. The empirical techniques allow for fairly arbitrary time variation in risk premiums. We find that the APT provides a better description of the expected returns on assets than the capital asset pricing model (CAPM). However, some statistically reliable mispricing of assets by the APT remains.

1. Introduction

In this paper we estimate and test the restrictions implied by an equilibrium version of Ross's arbitrage pricing theory (APT). We estimate the return factors using the asymptotic principal components technique first suggested by Chamberlain and Rothschild (1983) and extended by Connor and Korajczyk (1986). We test the cross-sectional restrictions imposed by the APT with a variety of multivariate procedures.

Section 2 describes the APT specification that we test. We use both the standard, static version of the APT and an intertemporal version developed in Connor and Korajczyk (1987). In this second version there is one factor that has a unit beta for every security. The static version does not impose this unit-beta restriction.

*We would like to thank Sankarshan Acharya, Gary Chamberlain, Lawrence Harris, Robert Hodrick, Ravi Jagannathan, Donald Keim, Allan Kleidon, Bruce Lehmann, Craig MacKinlay, Robert McDonald, Jay Shanken, Daniel Siegel, an anonymous referee, and the editor, John Long, for generous comments and suggestions. The paper also benefited from discussions with the seminar participants at University of Alberta, Carnegie-Mellon University, University of Chicago, INSEAD, University of Minnesota, Northwestern University, University of Pennsylvania, Southern Methodist University, Stanford University, and University of Southern California. The usual caveat applies.

In section 3 we outline the asymptotic principal components technique that we use to estimate the pervasive economic factors and a new iterative version that is more efficient than the one-step procedure described in Connor and Korajczyk (1986). These factor estimates are valid in a model with time-varying risk premiums, as long as asset betas are constant. We estimate the return factors and relate them to some macroeconomic time series suggested as possible sources of pervasive economic risk by Chen, Roll, and Ross (1986). We also analyze the estimation error in the factors using our technique and discuss the relationship between our method and standard factor analysis.

In section 4 we describe our testing procedures and empirical results. We use large cross-sectional samples (between 1,487 and 1,745 firms), both grouped into size-based portfolios and at the individual security level, to test the model. We perform tests using the disaggregated data by placing prior restrictions on the covariance matrix of residuals. The techniques are also applied to the CAPM, using standard proxies for the market portfolio. The APT explains the anomalous size-related seasonal patterns in returns that have been documented by others, although some nonseasonal anomalies persist. We conclude with a summary and suggestions for extensions.

2. Empirical specification of the APT

We briefly describe the asset pricing model to be tested. More detailed discussion of the APT can be found in Ross (1976), Chamberlain and Rothschild (1983), and Connor (1984). Let r denote the countably infinite vector of returns to a countably infinite set of traded assets. Assume that asset returns follow an approximate factor model,

$$\begin{aligned} &\tilde{r}_t = \mathrm{E}(\tilde{r}_t) + B\tilde{f}_t + \tilde{\varepsilon}_t, \\ &\mathrm{E}(\tilde{\varepsilon}_t | f_t) = 0, \quad \mathrm{E}\left(\tilde{f}_t\right) = 0, \quad \mathrm{E}(\tilde{\varepsilon}_t \tilde{\varepsilon}_t') = V, \end{aligned} \tag{1}$$

where $\tilde{f}_t$ is a k-vector of pervasive economic factors, B is an $\infty \times k$ matrix of the factor sensitivities of the assets, and $\tilde{\varepsilon}_t$ is the vector of idiosyncratic returns.

Let B^n denote the first n rows of B and V^n denote the first n rows and columns of V. Let $\|\cdot\|$ denote the matrix L^2-norm.[1] Assume that

$$\left\| \left(\frac{1}{n} B^{n\prime} B^n \right)^{-1} \right\| < c_1 < \infty \quad \text{for all } n,$$

$$\|V^n\| < c_2 < \infty \qquad \text{for all } n,$$

[1]For a square matrix X, $\|X\| = \max |g'Xg|$, subject to $g'g = 1$.

and that there is a cross-sectional average idiosyncratic variance

$$\sigma^2 = \operatorname*{plim}_{n \to \infty} \frac{1}{n} \tilde{\varepsilon}_t^{n\prime} \tilde{\varepsilon}_t^n,$$

where plim denotes the limit in probability. The equilibrium version[2] of the APT implies that

$$\mathrm{E}(\tilde{r}_t) = r_{Ft} e + B\gamma_t, \tag{2}$$

where r_{Ft} represents the return on a riskless asset, e is a vector of ones, and γ_t is a k-vector of factor risk premiums.

Combining equations (1) and (2) gives

$$\tilde{r}_t - r_{Ft} e = B\left(\gamma_t + \tilde{f}_t\right) + \tilde{\varepsilon}_t. \tag{3}$$

The relation (3) provides us with the basis for testing the restrictions implied by the model.

Let r^n denote an $n \times T$ matrix consisting of the observed returns on n assets over T periods. Let r_F denote a T-vector of observed returns on the riskless asset. The $n \times T$ matrix of excess returns (returns in excess of the riskless return) is given by $R^n = r^n - e^n r_F'$. Using (3) we can write the excess returns as

$$R^n = B^n F + \varepsilon^n, \tag{4}$$

where F is the $k \times T$ matrix of realizations of $(\gamma_t + f_t)$ over the period and ε^n is the $n \times T$ matrix of realizations of ε_t. In the empirical specification of the APT used here we allow for time variation in factor risk premiums (γ_t) but assume that factor sensitivities (B^n) are time-invariant.

3. Statistical identification of the factors

In Connor and Korajczyk (1986) we describe a new technique for identifying statistically the pervasive factors, plus their associated risk premiums, assumed by the APT. We call this approach *asymptotic principal components*. It is similar to standard principal components except that it relies on asymptotic results as the number of cross-sections grows large. In this section, we briefly review the relevant results from that paper, develop a more efficient

[2]The distinction between the standard APT and equilibrium versions of the APT is that the standard arbitrage conditions imply that (2) holds as an approximation whereas the equilibrium models [e.g., Connor (1984)] use additional restrictions on tastes and supplies of assets to derive (2) as an equality. See Shanken (1985b) for a discussion of this distinction.

version of the estimator, and show that the technique is valid for models with time-varying risk premiums. In addition we compare our estimated factors with some standard market indices and a set of macroeconomic time series suggested as sources of pervasive economic risk by Chen, Roll, and Ross (1986).

3.1. Asymptotic principal components

Denote the $T \times T$ cross-product matrix $\Omega^n = (1/n)R^{n\prime}R^n$. We apply a result from Connor and Korajczyk (1986) about the eigenvectors of Ω^n. Let G^n denote the orthonormal $k \times T$ matrix consisting of the first k eigenvectors of Ω^n. We show that G^n is approximately a nonsingular linear transformation of F.

Theorem 1. $G^n = L^n F + \phi^n$, *where* L^n *is a nonsingular matrix for all* n *and* $\operatorname{plim}_{n \to \infty} \phi^n = 0$, *the zero matrix.*

Theorem 1 is based on the result from Chamberlain and Rothschild (1983) that, as the number of cross-sections grows large, eigenvector analysis is asymptotically equivalent to factor analysis. Note that we can determine F only up to a nonsingular linear transformation, L^n – this reflects the 'rotational indeterminacy' of factor models.

A simple example may be useful in providing some intuition for this result. The simplest case with which we can deal has one pervasive factor and two time periods, i.e.,

$$\tilde{R}_{it} = b_i(\gamma_t + \tilde{f}_t) + \varepsilon_{it}, \qquad i = 1,2,3,\dots, \quad t = 1,2.$$

Note that the risk premiums, γ_t, can vary through time arbitrarily, but are not separately identifiable from the mean-zero factor realization, $\tilde{f}_t$. In the example Ω^n is a 2×2 matrix whose (t, τ) element is equal to

$$\sum_{i=1}^{n} R_{it} R_{i\tau}/n.$$

The diagonal elements are given by

$$\Omega^n_{\tau\tau} = (\gamma_\tau + \tilde{f}_\tau)^2 \left(\sum_{i=1}^{n} b_i^2/n \right) + \left(\sum_{i=1}^{n} \varepsilon_{i\tau}^2/n \right)$$
$$+ 2(\gamma_\tau + \tilde{f}_\tau) \left(\sum_{i=1}^{n} b_i \varepsilon_{i\tau}/n \right), \qquad \tau = 1,2. \tag{5}$$

The off-diagonal terms in Ω^n are given by

$$\Omega_{12}^n = \Omega_{21}^n = (\gamma_1 + \tilde{f}_1)(\gamma_2 + \tilde{f}_2)\left(\sum_{i=1}^{n} b_i^2/n\right) + \left(\sum_{i=1}^{n} \varepsilon_{i1}\varepsilon_{i2}/n\right) + (\gamma_1 + \tilde{f}_1)\left(\sum_{i=1}^{n} b_i\varepsilon_{i2}/n\right) + (\gamma_2 + \tilde{f}_2)\left(\sum_{i=1}^{n} b_i\varepsilon_{i1}/n\right). \tag{6}$$

Under our assumptions, the (cross-sectional) average squared beta converges (as $n \to \infty$) to some value, say $\bar{b}^2$, and the (cross-sectional) average $\varepsilon_{i\tau}^2$ converges to σ^2. By the assumption of an approximate factor structure and temporally independent ε's, the last term in (5) and the last three terms in (6) converge (again as $n \to \infty$) to zero. Therefore, as $n \to \infty$, Ω^n converges to

$$\Omega = \bar{b}^2 \begin{bmatrix} (\gamma_1 + \tilde{f}_1)^2 & (\gamma_1 + \tilde{f}_1)(\gamma_2 + \tilde{f}_2) \\ (\gamma_1 + \tilde{f}_1)(\gamma_2 + \tilde{f}_2) & (\gamma_2 + \tilde{f}_2)^2 \end{bmatrix} + \sigma^2 I_2. \tag{7}$$

The limit matrix, Ω, contains all of the information we seek, [i.e., $(\gamma_1 + \tilde{f}_1)$ and $(\gamma_2 + \tilde{f}_2)$]. We merely need a means of extracting this information. The reader can check that the first eigenvector of Ω is proportional to the vector of realized factors plus their risk premiums.

3.2. New extensions of the technique

Here we offer a refinement, in terms of estimation efficiency, to our asymptotic principal components technique and show that the factor estimates allow for time-varying risk premiums.

We motivate our refinement by considering a well-known relationship between factor analysis and standard principal components analysis. Let Σ denote the true (not estimated) covariance matrix of returns and assume that they obey a strict factor model:

$$\Sigma = BB' + V, \tag{8}$$

where V is assumed to be a diagonal matrix. This is the model used in factor analysis. Pre- and post-multiply both sides of (8) by $V^{-1/2}$ to get

$$\Sigma^* = B^*B^{*\prime} + I, \tag{9}$$

where $B^* = V^{-1/2}B$, and $\Sigma^* = V^{-1/2}\Sigma V^{-1/2}$ which is the covariance matrix of the transformed asset returns $r^* = V^{-1/2}r$. Note that the principal components of (9) are identical to the factor loadings of (8) up to a nonsingular $k \times k$ transformation. That is, if we scale each asset return by the standard deviation of its idiosyncratic return, the principal components are identical to the factor loadings. This same scaling can improve the efficiency of our asymptotic principal components technique. Recall that our original procedure begins with the $n \times T$ matrix of asset excess returns R^n and the cross-product matrix $\Omega^n = R^{n\prime}R^n/n$, and identifies the factors G^n as the first k eigenvectors of Ω^n. We have shown that G^n approaches a nonsingular transformation of the true factors F.

The following variant of our procedure also yields the true factors asymptotically but converges more quickly. Let $DIAG(V)$ denote the matrix whose (i, i) element is the ith diagonal element of the covariance matrix of idiosyncratic returns and whose (i, j) element is 0 for $i \neq j$. Construct the scaled matrix of excess returns $R^* = DIAG(V)^{-1/2}R$ and the corresponding cross-product matrix Ω^*. As long as the eigenvalues of V^n are bounded (we do not require that V^n is diagonal), it is easy to show[3] that the eigenvectors of Ω^* will converge to a nonsingular transformation of the factors. In most cases these eigenvectors will converge more quickly than will the eigenvectors of the cross-product matrix of unscaled returns, because the idiosyncratic components of the scaled asset returns have identical (unit) variances across assets. The procedure is analogous to the use of weighted least squares in a regression model.

The procedure is implemented as follows. First estimate the factors by calculating G^n, the eigenvectors of Ω^n. Then estimate the diagonal elements of V^n by calculating the residual variance of a regression of R^n on G^n (plus a constant). Calculate $\Omega^{n*} = (1/n)R^{n*\prime}R^{n*}$ and reestimate G^{n*}. Empirically, our sample sizes are sufficiently large that G^{n*} does not provide much improvement over G^n. Applications with smaller cross-sectional samples may find greater improvement. Note, also, that we must use estimates of the idiosyncratic variances, whereas our proof assumes knowledge of the true idiosyncratic variances. Since we are allowing n to approach infinity, with T fixed, we cannot rely on the standard T-consistency of $\hat{V}$. This estimation risk may reduce the efficiency gain of the procedure.

Recent empirical work suggests that asset risk premiums vary through time [see, e.g., Brown, Kleidon, and Marsh (1983), Keim (1983), Keim and Stambaugh (1986), and Ferson, Kandel, and Stambaugh (1987)]. The analysis in section 3.1 assumes that asset returns follow an exact multi-factor asset

[3] The proof is available from the authors. In it we use a slightly stronger assumption about the mean square idiosyncratic returns. Let $z_i = \varepsilon_i^2 - V_{ii}$, $z^n = (z_1, \ldots, z_n)'$, and $Q^n = E[z^n z^{n\prime}]$. In place of plim $\varepsilon^{n\prime}\varepsilon^n/n = \sigma^2$ we assume that $\|Q^n\|$ is bounded for all n.

pricing model and that B is constant, but places no restrictions on the time series properties of the factor-risk premiums, γ_t. Thus, our factor estimation procedure requires time-constant betas but allows time variation in risk premiums. In fact, $\tilde{f}_t$ and γ_t are not separately identified. Empirically, we find temporal variation in the estimated risk premiums. In particular, there are strong seasonalities. There is a related finding in Chan, Chen, and Hsieh (1985).

Although allowing for variation in the risk premiums is consistent with observed returns, the factor model approach does not link the temporal variation to the underlying 'primitive' parameters of the model (e.g., variation in expected marginal utility ratios from an optimally chosen consumption stream).

3.3. *Empirical properties of factor estimates*

In this section we compare our estimated factors with standard stock market portfolio proxies and interest rate variables. We also provide simulation evidence on the accuracy of the asymptotic principal components technique in an approximate factor model environment.

We estimate the factors and risk premiums using monthly stock returns in four nonoverlapping five-year subperiods, 1964–1968, 1969–1973, 1974–1978, and 1979–1983. The choice of five-year intervals makes our results comparable to earlier work such as Black, Jensen, and Scholes (1972) and Gibbons (1982). We estimate the factors by applying asymptotic principal components to the entire sample of New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) firms with no missing observations over the five-year subperiod. The numbers of firms available are 1,487, 1,720, 1,734, and 1,745, respectively, and the number of time periods is 60 for each subperiod.[4] The riskless return is assumed to be equal to the return on Treasury bills taken from Ibbotson Associates (1985).

To get an understanding of the behavior of the factors in relation to standard market portfolios we regress the excess return on the equal-weighted and value-weighted CRSP (Center for Research in Security Prices) portfolios on the first factor, the first five factors, and the first ten factors. To facilitate comparisons across indices we scale the factors so that the equal-weighted CRSP portfolio has betas equal to 1.0. The estimated intercept term and the R^2 values are invariant to this type of rescaling. The results for the one-factor and five-factor regressions are shown in table 1. An interesting feature of these regressions is that the first factor generally explains over 99% of the variance

[4]The requirement that firms have no missing observations eliminates about 30% of the total CRSP universe of NYSE and AMEX firms. The average number of firms with returns observed in a given month is 2,151, 2,474, 2,567, and 2,341 in the four subperiods, respectively.

Table 1

Regression of monthly market index returns, in excess of one-month Treasury bill returns, on factors estimated by asymptotic principal components. Indices are the value-weighted (*VW*) and equal-weighted (*EW*) portfolios of NYSE and AMEX stocks from CRSP, a low-grade bond portfolio (*JBRET*), and a long-term government bond portfolio (*UTS*). Factors, G_{jt}, are estimated using monthly stock returns on 1,487, 1,720, 1,734, and 1,745 securities over the periods 1964–1968, 1969–1973, 1974–1978, and 1979–1983, respectively.

(A) One-factor model $\tilde{R}_{it} = a_i + \beta_{i1}G_{1t} + \varepsilon_{it}$

Index[a]	$a_i \times 1200$	β_{i1}	R^2	$\bar{R}^2$
		1964–1968		
VW	−4.68	0.54	0.652[b]	0.647[c]
EW	0.11	1.00	0.983	0.982
JBRET	−4.08	0.08	0.074	0.058
UTS	−4.83	0.01	0.002	−0.016
		1969–1973		
VW	4.44	0.60	0.816	0.813
EW	−0.40	1.00	0.995	0.995
JBRET	1.13	0.17	0.276	0.264
UTS	0.92	0.09	0.049	0.032
		1974–1978		
VW	−7.20	0.55	0.651	0.645
EW	1.00	1.00	0.993	0.993
JBRET	−2.64	0.20	0.400	0.390
UTS	−1.52	0.07	0.079	0.063
		1979–1983		
VW	−4.56	0.76	0.848	0.845
EW	0.02	1.00	0.994	0.994
JBRET	−2.52	0.24	0.230	0.217
UTS	−7.41	0.27	0.107	0.091

Table 1 (continued)

(B) Five-factor model $R_{it} = a_i + \beta_{i1}G_{1t} + \cdots + \beta_{i5}G_{5t} + \varepsilon_{it}$

Index[a]	$a_i \times 1200$	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}	R^2	$\bar{R}^2$
				1964–1968				
VW	−2.64	0.53	1.49	3.65	2.77	2.71	0.897[b]	0.887[c]
EW	0.79	1.00	1.00	1.00	1.00	1.00	0.994	0.993
JBRET	−2.28	0.07	−0.15	−0.34	1.82	−2.95	0.354	0.295
UTS	−3.24	0.00	−1.30	−0.63	1.82	−0.55	0.181	0.106
				1969–1973				
VW	0.85	0.59	5.14	−9.60	7.13	−3.55	0.943	0.938
EW	−1.20	1.00	1.00	1.00	1.00	1.00	0.997	0.997
JBRET	0.67	0.17	0.83	−19.44	−11.26	−2.03	0.396	0.340
UTS	4.49	0.10	1.01	13.07	−30.37	−2.22	0.351	0.291
				1974–1978				
VW	−5.52	0.55	6.06	−3.56	−5.09	2.83	0.966	0.962
EW	0.56	1.00	1.00	1.00	1.00	1.00	0.998	0.998
JBRET	−2.52	0.20	2.09	−0.85	1.09	87.73	0.591	0.553
UTS	−0.63	0.07	1.31	−2.03	−1.62	−2.16	0.262	0.194
				1979–1983				
VW	−2.72	0.76	−3.86	7.54	−0.03	−1.59	0.946	0.941
EW	0.06	1.00	1.00	1.00	1.00	1.00	0.998	0.997
JBRET	−2.28	0.24	4.27	6.01	4.28	2.85	0.523	0.479
UTS	−6.85	0.27	8.40	12.59	6.11	5.46	0.485	0.437

[a]The factor estimates, G_{jt}, are scaled so that the equal-weighted CRSP portfolio has a unit beta for each factor. This scaling has no effect on the estimates of a_i or R^2.

[b]Unadjusted R^2.

[c]Adjusted R^2.

of the equal-weighted portfolio. The remaining factors have statistically significant explanatory power (see table 2) but obviously explain much less of the variance. For the value-weighted portfolio the results are quite different. The first factor still explains most of the variance of the portfolio, but much less than it does for the equal-weighted portfolio. The additional factors, again, are important. However, even with ten factors we do not reach an R^2 value obtained in the relation of the equal-weighted portfolio with just one factor.[5]

Table 1 also includes the results of regressing the excess returns of a portfolio of bonds with ratings below Baa (denoted *JBRET*) and the excess returns on long-term government bonds (denoted *UTS*) on the factor estimates. These data are from Ibbotson (1979) and Ibbotson Associates (1985), respectively. Similar variables were found to be important factors (in explaining cross-sectional differences in mean returns) by Chen, Roll, and Ross (1986). For *JBRET* we use returns in excess of the riskless rate, whereas their variable *UPR* uses returns in excess of the return on an Aaa bond portfolio. The first factor explains between 7% and 40% of the variance of the junk bond returns and the first five factors explain between 35% and 59% of the variance. The sixth through tenth factors do not have significant explanatory power.

The factors explain less of the variability in the excess returns on long-term government bonds than they do for the other indexes. The first factor explains between 0% and 11% of *UTS* and the first five factors explain between 18% and 49% of the variation. The sixth through tenth factors do not have significant explanatory power except in the 1974–1978 subperiod.

The high correlation between our factor estimates and the stock and bond market indices is not sufficient to guarantee that we will pick up the same cross-sectional pricing relation as Chen, Roll, and Ross (1986). However, lack of correlation might indicate that our factor estimates omit important priced factors. Thus, we view the correlations in table 1 as encouraging in the sense that a necessary (but not sufficient) condition for consistency with Chen, Roll, and Ross is met.

Some previous empirical studies have drawn inferences about the validity of the APT by testing whether the estimated factor risk premiums are different from zero, on average. Although this is not the approach we take, individual and joint tests of whether the unconditional means of the factors are equal to zero are presented in panel A of table 3 for the sake of comparison with earlier work. Equivalent tests are also shown for the equal-weighted stock portfolio. The test that the means of the first five factors are jointly zero (last column) is significant at the 10% level in the first two subperiods and not significant in the last two. Aggregating across the four subperiods yields a statistic that is significant at the 10% level. In general, the results in panel A of table 3 seem to

[5] We estimate these same regressions using factors estimated by the iterative procedure described above. Note that in calculating Ω^{n*} each security is weighted inversely proportional to its idiosyncratic variance. This will tend to place less weight on small firms in relation to large firms. Since the results are virtually identical to the results in table 1, we do not report them here.

Table 2

Wald test statistics and p-values for tests of explanatory power of additional factors in time series regressions of monthly market index excess returns on factors estimated by asymptotic principal components. Factors are estimated using monthly stock returns on 1,487, 1,720, 1,734, and 1,745 securities over the periods 1964–1968, 1969–1973, 1974–1978, and 1979–1983, respectively. One-factor vs. five-factor test is a joint test that the second through fifth factors have no explanatory power in a regression of the index on the first five factors. Five-factor vs. ten-factor test is a joint test that the sixth through tenth factors have no explanatory power in a regression of the index on the first ten factors.

	Test			
	One factor vs. five factors $H_0: \beta_{i2} = \cdots = \beta_{i5} = 0$		Five factors vs. ten factors $H_0: \beta_{i6} = \cdots = \beta_{i10} = 0$	
Index	$F_{4,54}$[a]	(p-value)[b]	$F_{5,49}$[a]	(p-value)[b]
	1964–1968			
Value-weighted stocks	32.01	(< 0.001)	2.96	(0.02)
Equal-weighted stocks	23.23	(< 0.001)	3.64	(0.01)
Low-grade corp. bonds	5.86	(0.001)	1.29	(0.28)
Long-term govt. bonds	2.97	(0.03)	0.69	(0.64)
	1969–1973			
Value-weighted stocks	30.35	(< 0.001)	2.72	(0.03)
Equal-weighted stocks	10.55	(< 0.001)	5.14	(0.001)
Low-grade corp. bonds	2.67	(0.04)	0.95	(0.46)
Long-term govt. bonds	6.31	(< 0.001)	0.68	(0.64)
	1974–1978			
Value-weighted stocks	123.61	(< 0.001)	1.53	(0.20)
Equal-weighted stocks	30.51	(< 0.001)	3.38	(0.01)
Low-grade corp. bonds	6.28	(< 0.001)	0.73	(0.61)
Long-term govt. bonds	3.36	(0.02)	3.15	(0.02)
	1979–1983			
Value-weighted stocks	24.69	(< 0.001)	3.68	(0.01)
Equal-weighted stocks	19.00	(< 0.001)	2.42	(0.05)
Low-grade corp. bonds	8.28	(< 0.001)	1.99	(0.10)
Long-term govt. bonds	9.90	(< 0.001)	1.14	(0.35)

[a] Wald test as in Theil (1971, p. 313). Statistic has an F distribution under the null hypothesis.
[b] p-value is the probability of obtaining a larger F statistic under the null hypothesis.

tell us more about the power (or lack of power) of tests involving unconditional means than about the value of the true means, since we reject a zero mean excess return for the equal-weighted stock portfolio with about the same frequency as for the factors.[6] We know that with a sufficiently long time series

[6] There are two common methods of performing the Hotelling T^2 test for joint hypotheses on mean vectors that calculate different functions of the same basic quadratic form. One version has a χ^2 distribution and the other an F distribution. Although equivalent asymptotically, the latter version is more conservative (leads to rejection less often) in small samples. We report the more conservative statistic.

Table 3

Test statistics and p-values (in parentheses) for the hypothesis that the unconditional mean factor risk premium is equal to zero and for the hypothesis that the conditional mean factor risk premium in January is equal to the unconditional mean factor risk premium. Test statistics are calculated for the first through fifth factors and CRSP equal-weighted market individually and for the first through fifth factors jointly. Factors are estimated using monthly stock returns in 1,487, 1,720, 1,734, and 1,745 securities over the periods 1964–1968, 1969–1973, 1974–1978, and 1979–1983, respectively.

Period	Equal-weighted market	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 1–5
(A) Tests for zero unconditional factor risk premiums							
1964–1968	10.18[a]	10.45[a]	1.29[a]	0.01[a]	1.37[a]	0.42[a]	2.78[c]
	(0.002)[b]	(0.002)	(0.261)	(0.905)	(0.246)	(0.521)	(0.026)
1969–1973	2.10	2.04	2.47	2.99	2.30	0.01	2.08
	(0.152)	(0.158)	(0.121)	(0.089)	(0.134)	(0.973)	(0.081)
1974–1978	1.7[illegible]	1.54	0.00	0.89	0.12	0.16	0.52
	[illegible]	[illegible]	(0.991)	(0.348)	(0.733)	(0.692)	(0.761)
1979–[illegible]	[illegible]	[illegible]	0.13	0.09	0.02	0.10	0.70
	[illegible]	(0.070)	(0.725)	(0.767)	(0.891)	(0.749)	(0.623)
(B) Tests for zero difference between January and unconditional factor premiums							
1964–1968	6.52[d]	7.57[d]	0.14[d]	7.18[d]	0.35[d]	3.03[d]	4.11[e]
	(0.013)	(0.008)	(0.712)	(0.010)	(0.556)	(0.087)	(0.003)
1969–1973	1.89	2.33	36.90	4.43	0.65	1.90	12.57
	(0.174)	(0.132)	(0.000)	(0.040)	(0.423)	(0.172)	(0.000)
1974–1978	18.06	18.90	5.93	2.47	6.92	0.01	10.52
	(0.000)	(0.000)	(0.018)	(0.121)	(0.011)	(0.923)	(0.000)
1979–1983	1.33	1.66	0.01	22.12	4.56	0.71	6.95
	(0.254)	(0.203)	(0.916)	(0.000)	(0.037)	(0.404)	(0.000)

[a] Hotelling T^2 test (distributed $F_{1,59}$) for the hypothesis H_0: $\mu_i = 0$.
[b] p-values in parentheses.
[c] Hotelling T^2 test (distributed $F_{5,55}$) for joint hypothesis H_0: $\mu_1 = \cdots = \mu_5 = 0$.
[d] Wald test (distributed $F_{1,58}$) for difference in mean return in January versus the rest of the year, H_0: $\mu_{iJ} = \mu_{iNJ}$.
[e] Modified likelihood ratio test [see Rao (1973 p. 555)] (distributed $F_{5,54}$) for joint hypothesis H_0: $\mu_{iJ} = \mu_{iNJ}$, $i = 1,\ldots,5$.

the equal-weighted portfolio will have a mean excess return different from zero.[7]

We test for seasonality in factor mean returns by regressing the factors on a constant and a dummy variable that is equal to one in January and zero otherwise. Seasonality is implied by non-zero coefficients on the dummy variable. The factors shown in panel B of table 3 exhibit significant seasonal

[7] For example, see Ibbotson Associates (1985).

differences in mean returns. This is consistent with some of the anomalous empirical evidence in relation to the CAPM. There is significant (at the 5% level) seasonality in at least half of the subperiods for each factor except for the fifth.

As Theorem 1 indicates, the asymptotic principal components estimates converge to a transformation of the factors, L^nF, as n approaches infinity. Obviously, it is useful to determine whether the actual number of securities used here is sufficiently large that we can ignore the estimation error, $\phi^n = G^n - L^nF$. To do this we present simulation results of asset return series that conform to an approximate factor model, estimate the pervasive factors by asymptotic principal components, and compare the factor estimates with the 'true' factors.

We use the first five estimated factors obtained from the 1979–1983 subperiod as the 'true' factors, F (F is a 5×60 matrix). The estimates of each asset's sensitivity to the factors and idiosyncratic variance are obtained from ordinary least squares (OLS) regressions of assets' excess returns on the factors. Let B denote the 1745×5 factor sensitivity matrix. The nondiversifiable component of asset returns is given by BF. Idiosyncratic returns are constructed to be temporally independent but possibly cross-sectionally dependent. The idiosyncratic return for asset i in period t is constructed as

$$\varepsilon_{it} = \rho\varepsilon_{i-1,t} + \eta_{it}, \qquad i = 2, \ldots, 1745,$$

$$\varepsilon_{1t} = \eta_{1t},$$

where η_{it} is a random drawing from a normal distribution with zero mean and a variance chosen so that $\sigma_i^2 = \operatorname{var}(\tilde{\varepsilon}_{it})$ is equal to the estimated idiosyncratic risks from the first-stage OLS regressions and $0 \leq \rho < 1$. The value of ρ determines the amount of nonfactor cross-sectional correlation in the sample. A value of $\rho = 0$ corresponds to the strict factor model studied originally by Ross (1976). One can show that

$$\lim_{n\to\infty} \|V^n\| \leq \max\left(\sigma_i^2\right) \cdot \frac{1+\rho}{1-\rho},$$

which is finite as long as $\rho < 1$ and the individual idiosyncratic variances are bounded. Thus the correlation structure corresponds to an approximate factor model as defined by Chamberlain and Rothschild (1983). Our asset return matrix is given by $BF + \varepsilon$, where ε is the 1745×60 matrix of residuals constructed in the above manner. Factor estimates for each iteration are given by the first five eigenvectors of $\Omega = R'R/n$. We compare these factor estimates with the 'true' factors by examining the R^2 values from the regression of each estimate on the five true factors. If there were no rotational indeterminacy we

Table 4

Simulation comparison of the asymptotic principal components factor estimates vs. true factors for a five-factor model (ten iterations). True factors, asset sensitivities (betas) relative to the first five factors, and idiosyncratic variances are estimated from monthly data on 1,745 firms in the 1979–1983 subperiod. Simulated returns are generated by adding a zero mean idiosyncratic return to the fitted factor-related return for each of the 1,745 assets. The parameter ρ determines the amount of idiosyncratic cross-correlation. The R^2 values are from regressing the estimated factor (column 2) on the five true factors. An R^2 value of 1.0 implies zero error in factor estimate. Average R^2 is the mean value of R^2 across the ten iterations (similarly for maximum and minimum R^2). Bias is average implied bias in estimated mispricing induced by assuming estimated factors are true factors (in basis points per annum).

ρ	Factor	Average R^2	Maximum R^2	Minimum R^2	Bias
0.0	1	0.997	0.997	0.996	−0.21
	2	0.986	0.989	0.980	
	3	0.977	0.986	0.969	
	4	0.965	0.977	0.948	
	5	0.960	0.972	0.937	
0.1	1	0.996	0.997	0.996	−0.14
	2	0.984	0.991	0.981	
	3	0980	0.986	0.971	
	4	0.966	0.973	0.957	
	5	0.958	0.967	0.951	
0.3	1	0.995	0.996	0.994	−0.19
	2	0.985	0.986	0.982	
	3	0.975	0.980	0.968	
	4	0.959	0.971	0.949	
	5	0.957	0.966	0.944	
0.5	1	0.993	0.994	0.992	−0.27
	2	0.981	0.988	0.976	
	3	0.970	0.980	0.962	
	4	0.958	0.971	0.942	
	5	0.949	0.960	0.934	
0.7	1	0.986	0.990	0.985	−0.43
	2	0.975	0.981	0.966	
	3	0.952	0.982	0.931	
	4	0.937	0.965	0.895	
	5	0.935	0.972	0.904	
0.8	1	0.980	0.984	0.978	−0.83
	2	0.966	0.981	0.950	
	3	0.930	0.965	0.884	
	4	0.915	0.953	0.848	
	5	0.875	0.924	0.825	
0.9	1	0.957	0.964	0.951	−1.49
	2	0.866	0.959	0.719	
	3	0.797	0.948	0.608	
	4	0.573	0.849	0.062	
	5	0.662	0.894	0.338	

could regress the first factor estimate on the first true factor. However, since G^n converges to L^nF rather than F, we must compare each of the factor estimates with the full set of true factors. In the limit (as $n \to \infty$) we would expect the R^2 value to be 1.0 for each regression.

Table 4 presents the simulation results for different values of ρ. The R^2 values are very large for all values of ρ, with the possible exception of $\rho = 0.9$ for factors 4 and 5. A value of $\rho = 0.9$ seems implausibly large, given that we have already extracted the first five factors. For the sake of comparison we calculate the average intraindustry idiosyncratic cross-correlation for every firm in our 1979–1983 subperiod. We use three-digit SIC codes to define industries and measure idiosyncratic returns in relation to a five-factor model. There were 1,684 firms in the 186 three-digit industries that had more than one firm. The average of the 15,364 intraindustry residual cross-correlations is 0.115. The bias parameter in the last column of table 4 is discussed in the next section. In general, the simulation results indicate that the asymptotic principal components technique provides accurate estimates of the pervasive economic factors.

4. Test results

In this section we test the restrictions implied by five-factor and ten-factor versions of the APT; we also test the CAPM using equal-weighted and value-weighted indices. The pricing theory of section 2 imposes a testable cross-equation restriction on the parameters of a multivariate regression of asset excess returns on the factors. Let a^n be the vector of intercepts in a regression of R^n on the factors

$$R^n = a^n e^{T\prime} + B^n F + \varepsilon^n, \tag{10}$$

(where B^n is the $n \times k$ matrix of asset betas). The pricing theory, as represented by eq. (4), implies that a^n should be identically zero. The parameter a^n is the APT analogue to Jensen's measure of abnormal performance [see Jensen (1968) and Jobson (1982)]. In our tests we replace F by the asymptotic principal components estimates of the factors, G. The rotational indeterminancy has no effect on the estimates of a^n and ε^n [see Connor and Korajczyk (1986, p. 383)]. We assume that the cross-sectional sample size used to estimate the factors is sufficiently large that the estimation error, ϕ^n, in Theorem 1 can be ignored.

Using G^n instead of F as the regressors in (10) induces an error in the variables (EIV) problem when ϕ^n is not identically zero. The asymptotic bias in a_i (as $T \to \infty$) implied by this EIV problem is given by

$$\operatorname*{plim}_{T\to\infty} (\hat{a}_i - a_i) = -(1 - \gamma' L^{n\prime} L^n \gamma)^{-1} \gamma' L^{n\prime} Q^n_\phi b_i,$$

where Q^n_ϕ is the $k \times k$ covariance matrix of factor estimation errors (ϕ^n) and b_i is the $k \times 1$ vector of factor sensitivities of asset i. The last column of table 4 gives estimates of the average bias in our simulations across the 1,745 assets. The average bias is expressed in basis points per annum (i.e., an entry of 1.0 represents an average bias of one hundredth of 1% per year). Our estimates of bias are extremely small in relation to the estimation error of a_i. Thus, we conclude that any rejection of the models tested below is not likely to be due solely to the use of the estimated factors, G^n, rather than the true factors, F.

We test for no mispricing ($a^n = 0$) against a general alternative hypothesis ($a^n \neq 0$) as well as some specific alternate hypotheses for size-related and seasonal effects. In addition to (10), we estimate the following model, which allows mispricing specific to the month of January to differ from mispricing that exists throughout the year [see Keim (1983)]:

$$R^n = a^n_{NJ} e^{T\prime} + a^n_J D_J' + B^n F + \xi^n, \tag{11}$$

where D_J is a T-vector that takes on the value of unity during January and zero elsewhere. The theory implies that $a^n_{NJ} = a^n_J = 0$ where a^n_{NJ} (a^n_J) represents the non-January (January) specific mispricing.

If asset returns follow a strict factor model in which idiosyncratic returns are independent across assets (i.e., V^n is diagonal) then joint tests of $a^n = 0$ would be relatively straightforward. In this case one would only need the estimates of a_i and the individual standard errors of the estimates. However, if asset returns follow only an approximate factor model (V^n nondiagonal with bounded eigenvalues as $n \to \infty$), we also need to calculate the covariances of $\hat{a}_i$ and $\hat{a}_j$ for $i \neq j$. Without prior restrictions, this requires the estimation and inversion of the full $n \times n$ covariance matrix $\hat{V}^n$. This is not feasible in our case, since n is between 1,487 and 1,745.

We use two approaches to overcome this problem. First, we group securities into portfolios on the basis of firm size, which has shown ability to predict deviations from the CAPM pricing relation, and test the hypothesis that the portfolio abnormal returns are zero. Such a grouping procedure, however, may mask important deviations from the model if the deviations are unrelated to the instruments used to assign assets into portfolios.

Because of the potential masking of pricing errors, we also test the model by estimating mispricing for each individual security. Tests of joint hypotheses about mispricing across assets are made feasible by the assumption that V^n is block-diagonal where the blocks are determined by three-digit SIC codes. That is, firms in different three-digit industries are assumed to have uncorrelated idiosyncratic returns.

We present the results for the grouped portfolios first and the disaggregated results later. In each subperiod we rank the securities with no missing observations by firm size and form ten portfolios. We define size as the market value of common equity the month before the beginning of the subperiod (e.g., for the 1964–1968 subperiod, size is calculated as market value at the end of December 1963). The first portfolio is an equal-weighted portfolio of firms from the smallest size decile, and so on. The ten time-series regressions of (10) [or of (11)] fit into a standard multivariate regression framework. The restrictions that $a^n = 0$ (or $a^n_{NJ} = a^n_J = 0$) can be tested with standard large-sample tests [e.g., Wald, likelihood ratio (LR), and Lagrange multiplier (LM) tests]. Although the tests are equivalent asymptotically, they may give conflicting results in small samples [Berndt and Savin (1977)]. In applications quite similar to ours Stambaugh (1982) shows that both the Wald and LR tests have pronounced tendencies to reject too often. Because of these problems the test statistics we report are modified versions of the LR test that are suggested in Rao (1973, pp. 554–556).[8] The statistic is given by

$$\left[\left(|\hat{V}_r|/|\hat{V}_u|\right) - 1\right] \cdot (T - k - p)/p, \tag{12}$$

where $|\hat{V}_r|(|\hat{V}_u|)$ is the determinant of the maximum likelihood estimate of the error covariance matrix from the regression in its restricted (unrestricted) form, T is the number of time series observations, k is the number of factors, and p is the number of cross-sections in the multivariate regression. For the hypotheses tested here, Rao (1973) shows that the statistic has (under the assumption of normally distributed errors) an exact small-sample distribution, which is $F(p, T - k - p)$. The statistic in (12) is identical to the statistic described on page 32 of Gibbons, Ross, and Shanken (1986). The use of this statistic or ones similarly adjusted for small samples can lead to quite different inferences from the usual large-sample test statistics [see Binder (1985) and Shanken (1985a)].

Table 5 gives the results of our tests of the pricing restrictions for the ten size-based portfolios. The APT does better (a lower frequency of rejections) than the CAPM in explaining the January seasonality in returns (testing $a^n_J = 0$). At first glance, the APT seems to do a worse job of explaining the nonseasonal mispricing ($a^n_{NJ} = 0$). Looking at the rejection rates across non-nested models can be misleading, however, since a model that actually fits better (smaller values of $|a|$) may be rejected if the deviations are measured more precisely (i.e., the test has more power).

[8] We have also calculated the standard versions of the Wald and LR statistics.

Table 5

Modified likelihood ratio (MLR) tests for the absence of mispricing using ten portfolios formed by market value.[a] Estimates of mispricing are obtained from a multivariate regression of monthly excess returns of ten portfolios (equal-weighted portfolios based on a ranking of market value at the beginning of each five-year subperiod) on (a) the excess returns on the equal-weighted and value-weighted CRSP portfolios of NYSE and AMEX stocks and (b) five and ten factors estimated by asymptotic principal components. The sample consists of 1,487, 1,720, 1,734, and 1,745 securities over the 1964–1968, 1968–1973, 1974–1978, and 1979–1983 subperiod, respectively. Unconditional mispricing is measured by the intercepts of the multivariate regression including a constant term. Conditional (seasonal) mispricing measured by the intercept and dummy slope coefficients in a multivariate regression including a constant term and a January dummy variable.

$$R^n = a^n e^{T\prime} + B^n G + \varepsilon^n \quad \text{and} \quad R^n = a^n_{NJ} e^{T\prime} + a^n_J D' + B^n G + \xi^n$$

		CAPM		APT	
Hypothesis	Period	Equal-weighted	Value-weighted	Five factors	Ten factors
$a^n = 0$	1964–83	3.82 (< 0.001)[b]	3.47 (< 0.001)		
$a^n_J = 0$	1964–83	8.24 (< 0.001)	11.24 (< 0.001)		
$a^n_{NJ} = 0$	1964–83	2.98 (0.001)	2.45 (0.009)		
$a^n = 0$	1964–68	1.70 (0.109)	2.62 (0.012)	2.62 (0.013)	2.41 (0.024)
$a^n_J = 0$	1964–68	0.70 (0.755)	1.52 (0.161)	0.85 (0.583)	0.37 (0.954)
$a^n_{NJ} = 0$	1964–68	1.56 (0.149)	2.11 (0.042)	2.34 (0.026)	2.12 (0.046)
$a^n = 0$	1969–73	1.82 (0.082)	1.72 (0.103)	1.93 (0.065)	1.75 (0.102)
$a^n_J = 0$	1969–73	6.40 (< 0.001)	6.14 (< 0.001)	1.17 (0.333)	1.09 (0.391)

$a^n_{NJ}=0$	1969–73	1.84 (0.078)	2.11 (0.042)	1.07 (0.408)	0.85 (0.588)
$a^n=0$	1974–78	0.83 (0.601)	1.13 (0.361)	1.73 (0.103)	2.02 (0.056)
$a^n_J=0$	1974–78	3.53 (0.001)	5.99 (< 0.001)	1.97 (0.061)	3.53 (0.002)
$a^n_{NJ}=0$	1974–78	0.64 (0.770)	0.71 (0.711)	1.74 (0.103)	2.02 (0.057)
$a^n=0$	1979–83	0.95 (0.495)	1.13 (0.358)	1.74 (0.100)	1.84 (0.085)
$a^n_J=0$	1979–83	2.66 (0.011)	2.99 (0.005)	0.87 (0.571)	0.63 (0.780)
$a^n_{NJ}=0$	1979–83	0.78 (0.649)	0.81 (0.618)	1.81 (0.087)	1.73 (0.108)
$a^n=0$	Aggregated[c]	50.06 (0.132)	60.44 (0.020)	71.63 (0.002)	70.99 (0.002)
$a^n_J=0$	Aggregated	102.17 (< 0.001)	123.71 (< 0.001)	46.05 (0.236)	48.37 (0.171)
$a^n_{NJ}=0$	Aggregated	45.70 (0.242)	52.99 (0.082)	63.19 (0.011)	60.56 (0.019)

[a] Independent variables are the CRSP stock portfolios or factor estimates produced by the asymptotic principal components technique, G, a vector of ones, e^T, and a dummy variable for January, D. Average mispricing is measured by a^n, January-specific mispricing by a^n_J, and non-January-specific mispricing by a^n_{NJ}. Test statistics are the modified LR test [see Rao (1973, p. 555)] which has an F distribution. Numerator degrees of freedom $(DF_1) = 10$. For the 1964–83 period denominator degrees of freedom (DF_2) is equal to 229 (a) and 228 $(a_J$ and $a_{NJ})$. For each subperiod DF_2 is equal to 49 (CAPM: a), 48 (CAPM: a_J and a_{NJ}), 45 (APT-5: a), 44 (APT-5: a_J and a_{NJ}), 40 (APT-10: a), and 39 (APT-10: a_J and a_{NJ}).

[b] p-values in parentheses.

[c] Aggregation of subperiod results. Statistic has χ^2 distribution, asymptotically, with 40 degrees of freedom under the null hypothesis.

A closer look at the parameter estimates gives a very different picture of the relative performance of the models than does a casual investigation of the test statistics in table 5. Fig. 1 plots the average mispricing (across the four subperiods) for each size portfolio in relation to the CAPM (using the value-weighted and equal-weighted portfolios of NYSE and AMEX stocks) and the APT with five factors. The vertical axis is the average value of a_i for each size portfolio from smallest (S1) to largest (S10). Although the aggregate statistics reported in table 5 indicate a stronger rejection of $a^n = 0$ for the APT than for the equal-weighted and value-weighted CAPM, the actual value of APT mispricing is much smaller (for all but one portfolio) in relation to the value-weighted CAPM and slightly smaller (for all but two portfolios) in relation to the equal-weighted CAPM. Thus, the stronger rejection of the five-factor APT is due to more precision in the estimates of a_i (i.e., R^2 values around 0.98 versus 0.75) rather than larger pricing errors.

Fig. 2 presents some of the most interesting results. One of the most persistent empirical anomalies in asset pricing has been that the common equity of small firms earns a much higher (risk-adjusted) return than the equity of large firms, particularly in January. The results in fig. 2 indicate that, when we use the five-factor APT to adjust for risk, there is no relation between January-specific mispricing and firm size. Also, although the APT does not totally explain the non-January specific size effect (see fig. 3), it does at least as well as the versions of the CAPM. Again, this is quite different from what one might guess from the statistics in table 5. The nature of the factor-model approach (in which the factor estimates are chosen to explain variation) would lead one to expect the APT might do better in explaining a time-varying size anomaly. It is encouraging, at least, that the evidence is consistent with a model in which the anomalous CAPM January seasonal effect is due to assets' different risks relative to factors with seasonal risk premiums. Work along the lines of Chen, Roll, and Ross (1986) may help us identify the nature of these seasonal factors.

Before we turn to the disaggregated results we present some evidence (in table 6) on whether the assumption of block diagonality, on the basis of three-digit SIC codes, is reasonable. Consider forming equal-weighted industry portfolios for each three-digit industry and estimating (10) for each industry. For each two-digit industry define V_{2D} as the error covariance matrix [from (10)] of the component three-digit portfolios. If returns are block-diagonal by three-digit industries, then V_{2D} is diagonal. To test whether V_{2D} is diagonal, define S_{2D} to be the ratio of the determinants of the unrestricted and restricted estimates of V_{2D}. An appropriate statistic for testing diagonality is [see Morrison (1976, pp. 258–259)]

$$\left[T - 1.5 - (j^2 - 1)/3(j - 1)\right] \ln S_{2D},$$

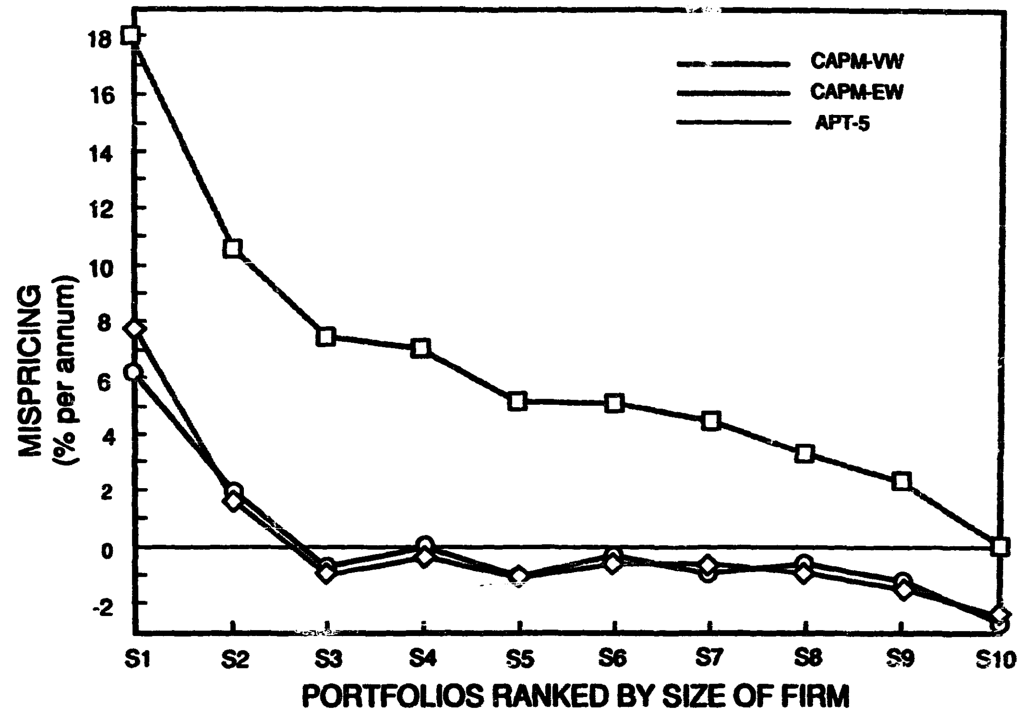

Fig. 1. Mispricing, in percent per annum, for ten portfolios formed by ranking on firm size. Size is defined as market value of common stock at the beginning of each subperiod. S1 represents the portfolio of smallest firms while S10 represents the portfolio of largest firms. For each of four subperiods (1964–1968, 1969–1973, 1974–1978, 1979–1983) mispricing is estimated by the intercept in the regression of monthly portfolio excess returns on a constant, and (a) monthly excess returns on the CRSP value-weighted portfolio of NYSE and AMEX stocks, denoted CAPM-VW (long dashes connecting squares), (b) monthly excess returns on the CRSP equal-weighted portfolio of NYSE and AMEX stocks, denoted CAPM-EW (short dashes connecting diamonds), and (c) first five-factor estimates from the asymptotic principal components procedure, denoted APT-5 (solid line connecting circles). Mispricing is the average mispricing across the four subperiods.

which, assuming normality, has an asymptotic distribution that is χ^2 with degrees of freedom equal to $(j^2-j)/2$, where j is the number of three-digit industries in the particular two-digit industry. Table 6 shows the number of two-digit industries that accept and reject the null at the 0.05 level. The industries marked N/A are those with only one three-digit industry within the two-digit classification.[9]

The results for the CAPM, especially using the value-weighted market portfolio, show a relatively large number of rejections. The five- and ten-factor models show less evidence against the block-diagonality assumption. There is

[9]These tests will tend to reject independence too often if security returns have probability distributions with larger kurtosis than the normal distribution [see Muirhead (1982, p. 547)]. Given the evidence of 'fat tails' in return distributions, the evidence against independence is not as strong as a literal interpretation of the numbers in table 6 would indicate.

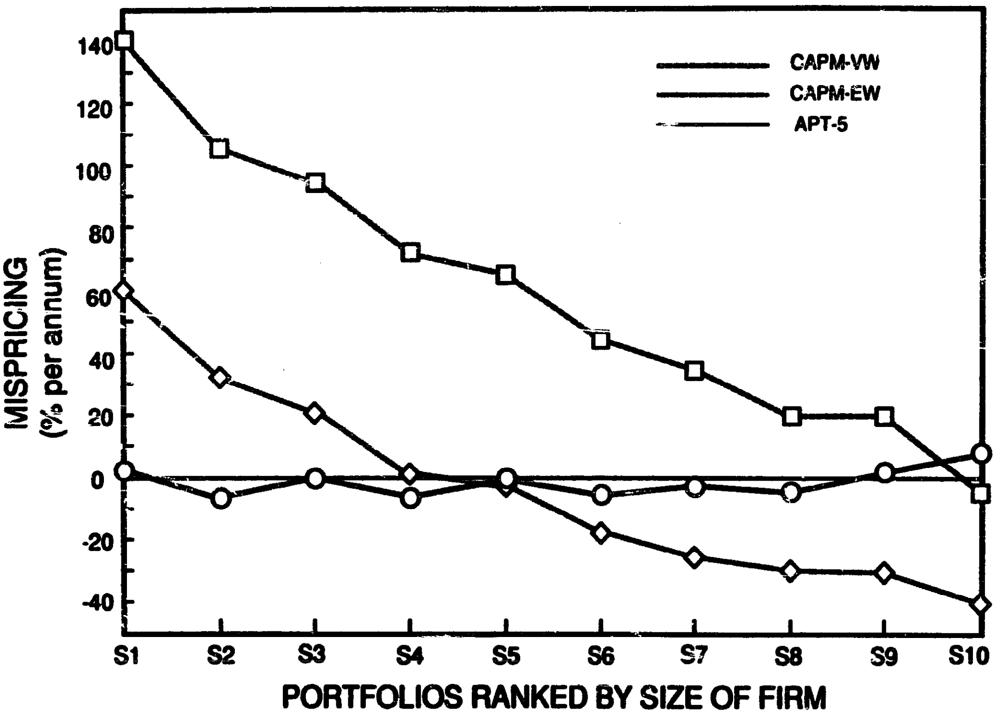

Fig. 2. January-specific mispricing, in percent per annum, for ten portfolios formed by ranking on firm size. Size is defined as market value of common stock at the beginning of each subperiod. S1 represents the portfolio of smallest firms, while S10 represents the portfolio of largest firms. For each of four subperiods (1964–1968, 1969–1973, 1974–1978, 1979–1983) mispricing is estimated by the slope coefficient on the January dummy variable in the regression of monthly portfolio excess returns on a constant, January dummy variable, and (a) monthly excess returns on the CRSP value-weighted portfolio of NYSE and AMEX stocks, denoted CAPM-VW (long dashes connecting squares), (b) monthly excess returns on the CRSP equal-weighted portfolio of NYSE and AMEX stocks, denoted CAPM-EW (short dashes connecting diamonds), and (c) first five-factor estimates from the asymptotic principal components procedure, denoted APT-5 (solid line connecting circles). Mispricing is the average mispricing across the four subperiods.

still some evidence in these models of nonindependence across three-digit industries.

Although the block-diagonality assumption is not strictly true, we believe it is a reasonable first step to using disaggregated data in testing the APT.[10] We also perform tests that assume V^n is diagonal. They are not reported here but are available from the authors.

The results of the tests for the disaggregated (nongrouped) regressions are presented in table 7. Under the assumption of block-diagonality by three-digit industries we can estimate the multivariate regressions (10) or (11) and calculate the test statistic (12) separately for each industry. If the block-

[10]A potential alternative approach would be to use relative firm size, rather than industry, to model the correlation structure. The results in Huberman and Kandel (1985) indicate that this approach may be fruitful.

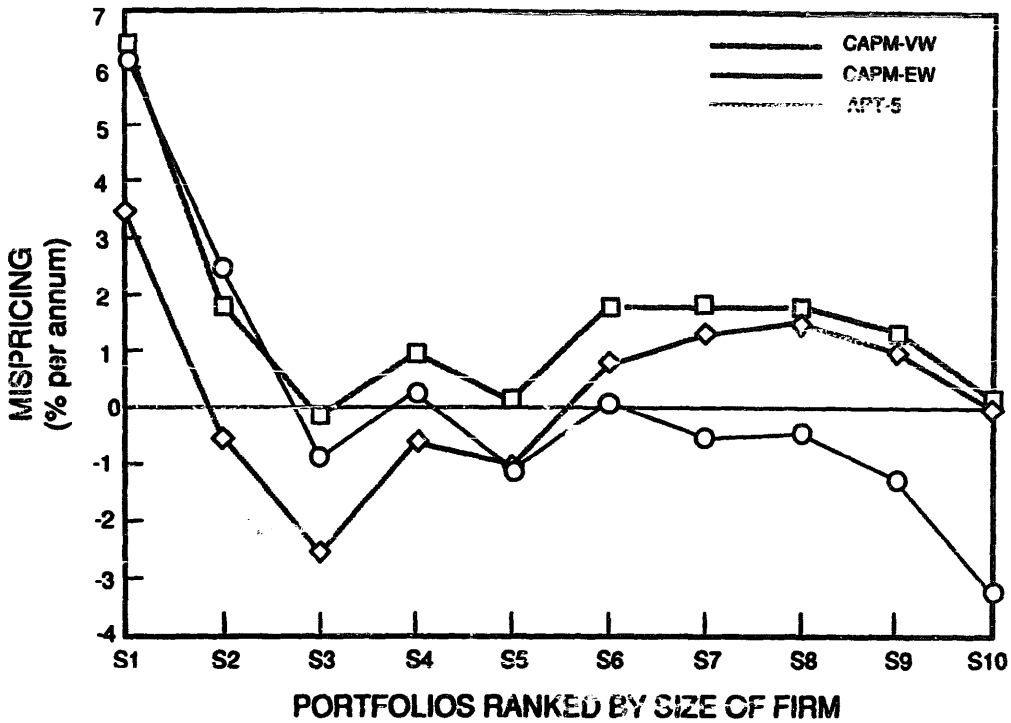

Fig. 3. Non-January-specific mispricing, in percent per annum, for ten portfolios formed by ranking on firm size. Size is defined as market value of common stock at the beginning of each subperiod. S1 represents the portfolio of smallest firms, while S10 represents the portfolio of largest firms. For each of four subperiods (1964–1968, 1969–1973, 1974–1978, 1979–1983) mispricing is estimated by the intercept in the regression of monthly portfolio excess returns on a constant, January dummy variable, and (a) monthly excess returns on the CRSP value-weighted portfolio of NYSE and AMEX stocks, denoted CAPM-VW (long dashes connecting squares), (b) monthly excess returns on the CRSP equal-weighted portfolio of NYSE and AMEX stocks, denoted CAPM-EW (short dashes connecting diamonds), and (c) first five-factor estimates from the asymptotic principal components procedure, denoted APT-5 (solid line connecting circles). Mispricing is the average mispricing across the four subperiods.

diagonality assumption is true, these F statistics are independent across blocks. However, unlike for χ^2 random variables, we cannot aggregate across blocks by simply summing the test statistics. We use an aggregation procedure similar to the one suggested, in a slightly different context, by Shanken (1985a).[11] We approximate each F statistic by a χ_p^2 distribution with the same tail area, where p is the number of cross-sections in the block. The sum of

[11]Shanken (1985a) suggests approximating the F statistic by a normal distribution. That is, find the value of a unit normal with the same tail area (p-value) as the computed F statistic. The sum of these unit normals, divided by the square root of the number of blocks, has a unit normal distribution. Our application is slightly different in that we are aggregating across blocks of different sizes (i.e., the F statistics have different degrees of freedom). Our method implicitly places greater weight on larger blocks, whereas use of the normal approximation places the same weight on each block regardless of size. This is not a concern for the tests in Shanken (1985a), since the F statistics in that study have the same degrees of freedom.

Table 6

Test results for block-diagonality of idiosyncratic covariance matrices where blocks are defined by three-digit SIC code industries.[a] 'Accept' represents the number of two-digit industries that fail to reject (at the 0.05 level) independence of the component three-digit idiosyncratic errors. 'Reject' represents the number of two-digit industries that reject (at the 0.05 level) independence. 'N/A' is the number of two-digit industries with only one three-digit industry.

(A) CAPM						
	Equal-weighted			Value-weighted		
Period	Accept	Reject	N/A	Accept	Reject	N/A
1964–68	23	24	14	9	38	14
1969–73	22	27	13	9	40	13
1974–78	20	31	12	16	35	12
1979–83	20	28	15	14	34	15

(B) APT						
	Five-factor			Ten-factor		
Period	Accept	Reject	N/A	Accept	Reject	N/A
1964–68	23	24	14	25	22	14
1969–73	28	21	13	32	17	13
1974–78	28	23	12	28	23	12
1979–83	27	21	15	27	21	15

[a]For each two-digit industry equal-weighted portfolios of each three-digit subindustry are formed. Idiosyncratic returns are measured by residuals from time series regressions of industry portfolio excess returns on market proxy excess return (CAPM) or factors (APT). Null hypothesis is that residual covariance matrix is diagonal. Assuming normality, test statistic is asymptotically χ^2 with degrees of freedom equal to $(j^2-j)/2$, where j is the number of three-digit industries within the two-digit industry. See Morrison (1976, pp. 258–259).

these χ^2 random variables has an asymptotic χ^2 distribution with degrees of freedom equal to the total number of cross-sections, n.[12]

The clearest inference that one can draw from the statistics in table 7 is that the hypothesis of no January mispricing ($a_J^n=0$) is rejected even when the APT is used as a benchmark. Over the second subperiod we reject the CAPM but not the APT in months other than January (testing $a_{NJ}^n=0$ at the 0.01 level). Overall, these test results seem to lack power to discriminate between the models. This is true especially in light of the potentially misleading inferences that might be drawn from exclusive reliance on the significance of the test statistics for nonnested models against general alternatives. It may be

[12]For the tests assuming block-diagonality, n is equal to 1,461, 1,687, 1,701, and 1,713, respectively. This is because one industry (electric utilities) has a very large number of firms in relation to the number of time series observations per period, hence $\hat{V}$ is singular or near singular. For this industry we formed portfolios consisting of two firms each. This reduced the number of cross-sections in this industry from 52, 66, 66, and 65 to 26, 33, 33, and 33 in each respective period.

Table 7

Modified likelihood ratio (MLR) tests for the absence of mispricing using individual securities.[a] Estimates of mispricing obtained from a multivariate regression of monthly security excess returns on (a) the excess returns on the equal-weighted and value-weighted CRSP portfolios of NYSE and AMEX stocks and (b) five and ten factors estimated by asymptotic principal components. The sample consists of 1,487, 1,720, 1,734, and 1,745 securities over the 1964–1968, 1968–1973, 1974–1978, and 1979–1983 subperiod, respectively. Unconditional mispricing is measured by the intercepts of the multivariate regression including a constant term. Conditional (seasonal) mispricing measured by the intercept and dummy slope coefficients in a multivariate regression including a constant term and a January dummy variable. Test statistics are calculated assuming the idiosyncratic covariance matrix is block-diagonal with blocks defined by three-digit SIC industrial codes.

$$R^n = a^n e^{T\prime} + B^n F + \varepsilon^n \quad \text{and} \quad R^n = a^n_{NJ} e^{T\prime} + a^n_J D' + B^n F + \xi^n$$

		CAPM		APT	
Hypothesis	Period	Equal-weighted	Value-weighted	Five factors	Ten factors
$a^n = 0$	1964–1968	1,083 (1.000)[b]	1,405 (0.847)	1,053 (1.000)	1,014 (1.000)
$a^n_J = 0$	1964–1968	1,965 (< 0.001)	2,279 (< 0.001)	1,738 (< 0.001)	1,589 (0.010)
$a^n_{NJ} = 0$	1964–1968	1,173 (1.000)	1,345 (0.985)	1,159 (1.000)	1,094 (1.000)
$a^n = 0$	1969–1973	1,494 (1.000)	1,655 (0.786)	1,261 (1.000)	1,277 (1.000)
$a^n_J = 0$	1969–1973	3,152 (< 0.001)	3,501 (< 0.001)	1,695 (0.441)	1,763 (0.096)
$a^n_{NJ} = 0$	1969–1973	2,006 (< 0.001)	2,332 (< 0.001)	1,411 (1.000)	1,423 (1.000)
$a^n = 0$	1974–1978	1,274 (1.000)	1,644 (0.883)	1,360 (1.000)	1,445 (1.000)
$a^n_J = 0$	1974–1978	2,763 (< 0.001)	4,269 (< 0.001)	2,128 (< 0.001)	1,740 (0.248)
$a^n_{NJ} = 0$	1974–1978	1,416 (1.000)	1,477 (1.000)	1,519 (0.999)	1,630 (0.890)
$a^n = 0$	1979–1983	1,098 (1.000)	1,249 (1.000)	1,141 (1.000)	1,161 (1.000)
$a^n_J = 0$	1979–1983	2,329 (< 0.001)	2,369 (< 0.001)	2,006 (< 0.001)	1,884 (0.002)
$a^n_{NJ} = 0$	1979–1983	1,256 (1.000)	1,312 (1.000)	1,342 (1.000)	1,367 (1.000)

[a]Independent variables are the CRSP stock portfolios or factor estimates produced by the asymptotic principal components technique, G, a vector of ones, e^T, and a dummy variable for January, D. Average mispricing is measured by a^n, January-specific mispricing by a^n_J, and non-January-specific mispricing by a^n_{NJ}. Within each block test statistics are the modified LR test [see Rao (1973, p. 555)] which has an F distribution. Test statistics are aggregated across industries. Aggregate test statistic has an asymptotic χ^2 distribution with degrees of freedom equal to 1,461 (1964–1968), 1,687 (1969–1973), 1,701 (1974–1978), and 1,713 (1979–1983).

[b]p-values in parentheses.

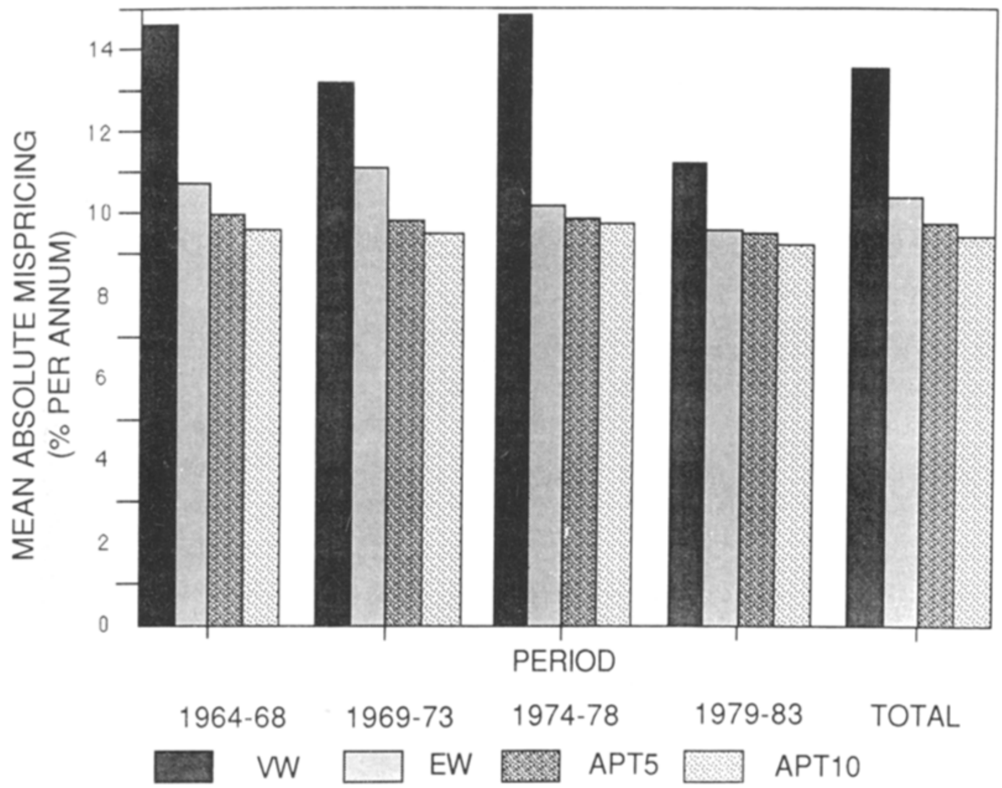

Fig. 4. Mean absolute mispricing, in percent per annum, for individual assets. For the four subperiods (1964–1968, 1969–1973, 1974–1978, 1979–1983) there are 1,487, 1,720, 1,734, and 1,745 assets, respectively. Mean absolute mispricing is estimated by the cross-sectional mean of the absolute value of the intercept in a regression of monthly individual asset excess returns on a constant, and (a) monthly excess returns on the CRSP value-weighted portfolio of NYSE and AMEX stocks, denoted VW, (b) monthly excess returns on the CRSP equal-weighted portfolio of NYSE and AMEX stocks, denoted EW, (c) first five-factor estimates from the asymptotic principal components procedure, denoted APT5, and (d) first ten-factor estimates from the asymptotic principal components procedure, denoted APT10. Total represents average absolute mispricing across the four subperiods.

that, even with the prior restrictions placed on the covariance matrix of the residuals, these tests have low power because of the large number of assets in relation to time series observations [see Gibbons, Ross, and Shanken (1986)].[13]

Figs. 4, 5, and 6 depict the mean absolute mispricing (MAM) of the individual assets for the alternative models. In general the APT has smaller MAM. It is not uncommon, however, for the ten-factor model to have larger MAM than the five-factor model. This may be due to estimation error of factor loadings for factors 6 through 10 that increases the error in $\hat{a}^n$ more than the decrease in the error of a^n due to including the risk premiums of additional factors (this includes the case where factors 6 through 10 are not priced).

[13]Also, the tendency of the p-values to cluster around zero or one indicates that assuming block-diagonality tends to understate the standard errors.

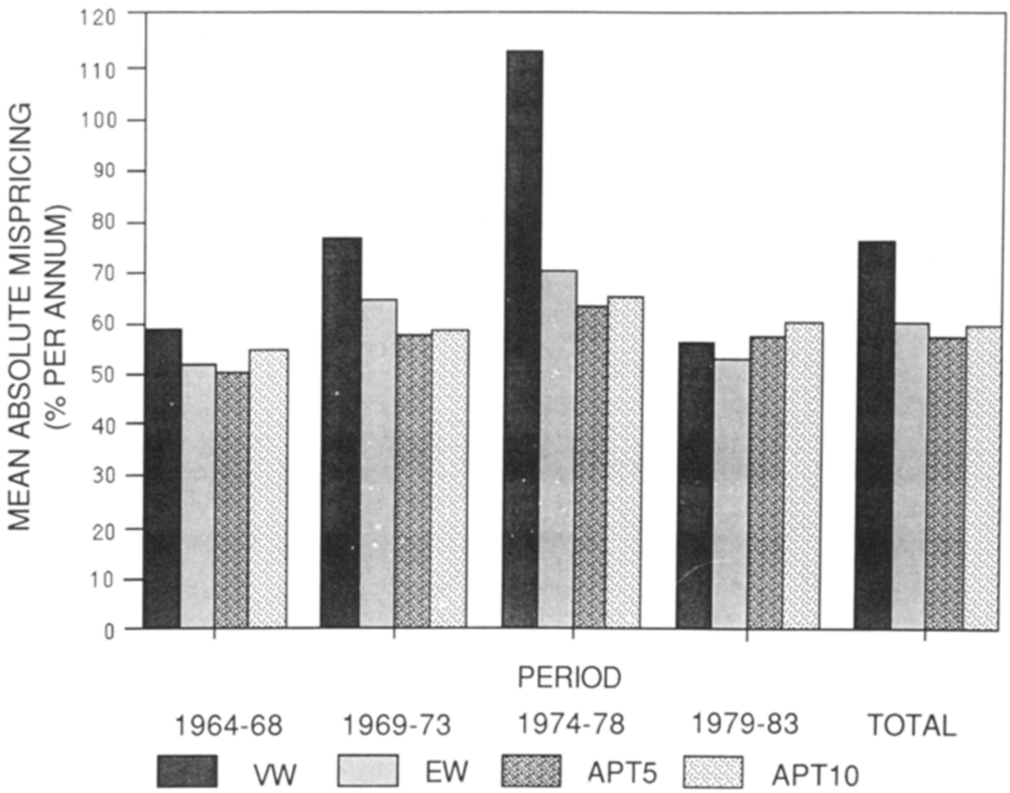

Fig. 5. Mean absolute January-specific mispricing, in percent per annum, for individual assets. For the four subperiods (1964–1968, 1969–1973, 1974–1978, 1979–1983) there are 1,487, 1,720, 1,734, and 1,745 assets, respectively. Mean absolute mispricing is estimated by the cross-sectional mean of the absolute value of the slope coefficient on the January dummy variable in a regression of monthly individual asset excess returns on a constant, a January dummy variable, and (a) monthly excess returns on the CRSP value-weighted portfolio of NYSE and AMEX stocks, denoted VW, (b) monthly excess returns on the CRSP equal-weighted portfolio of NYSE and AMEX stocks, denoted EW, (c) first five-factor estimates from the asymptotic principal components procedure, denoted APT5, and (d) first ten-factor estimates from the asymptotic principal components procedure, denoted APT10. Total represents average absolute January-specific mispricing across the four subperiods.

Given the mixed results when testing against the general alternative hypothesis of nonzero mispricing, we provide tests against the specific alternative that mispricing is related to the market capitalization of the firm. On the basis of Brown, Kleidon, and Marsh's (1983) result, we postulate a linear relation between abnormal returns and the logarithm of the value of each firm's common equity. We concentrate on the relations between a_J^n and a_{NJ}^n and firm size, represented as[14]

$$a_{iJ} = \omega_0 + \omega_1 LS_i, \tag{13a}$$

$$a_{iNJ} = \theta_0 + \theta_1 LS_i, \tag{13b}$$

[14] Results for a^n, from (10), are available from the authors.

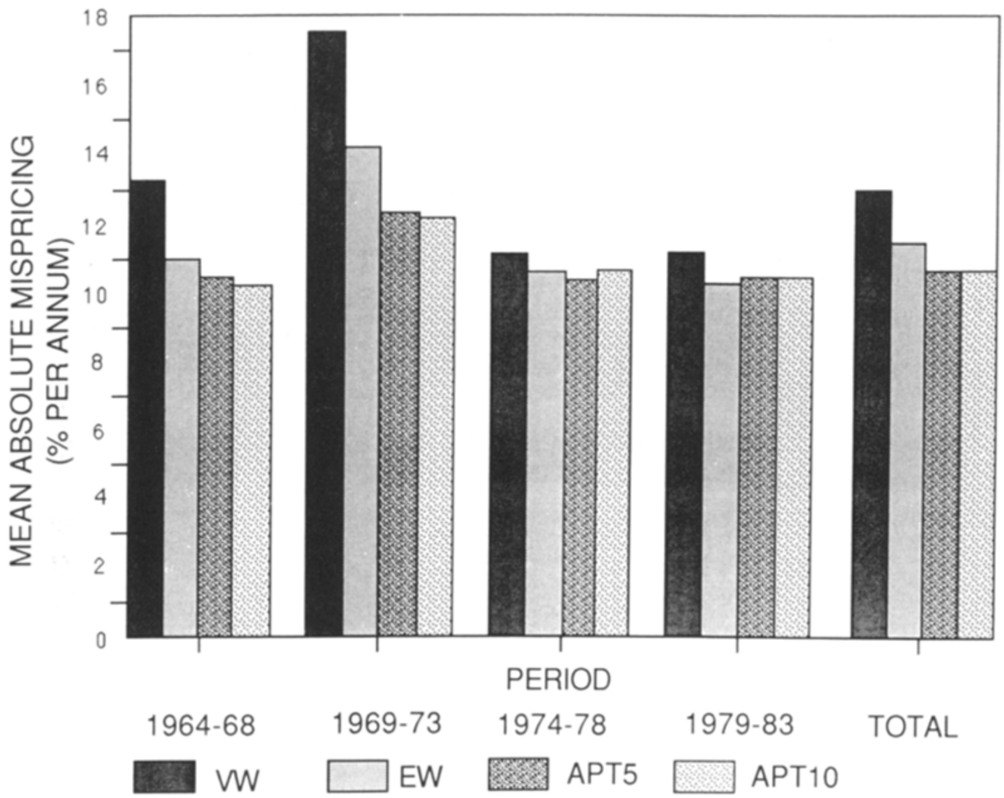

Fig. 6. Mean absolute non-January-specific mispricing, in percent per annum, for individual assets. For the four subperiods (1964–1968, 1969–1973, 1974–1978, 1979–1983) there are 1,487, 1,720, 1,734, and 1,745 assets, respectively. Mean absolute mispricing is estimated by the cross-sectional mean of the absolute value of the intercept in a regression of monthly individual asset excess returns on a constant, a January dummy variable, and (a) monthly excess returns on the CRSP value-weighted portfolio of NYSE and AMEX stocks, denoted VW, (b) monthly excess returns on the CRSP equal-weighted portfolio of NYSE and AMEX stocks, denoted EW, (c) first five-factor estimates from the asymptotic principal components procedure, denoted APT5, and (d) first ten-factor estimates from the asymptotic principal components procedure, denoted APT10. Total represents average absolute non-January-specific mispricing across the four subperiods.

for $i = 1, \dots, n$. Here, a_{iJ}, a_{iNJ}, and LS_i are mean 'abnormal' January-specific returns, mean 'abnormal' non-January-specific returns, and size (logarithm of capitalization) of firm i. Under the null hypothesis that the pricing model is correct, $\omega_0 = \omega_1 = \theta_0 = \theta_1 = 0$. We observe only estimates of a_{iJ} and a_{iNJ}, denoted $\hat{a}_{iJ}$ and $\hat{a}_{iNJ}$. Thus our tests involve estimating the relations

$$\hat{a}_{iJ} = \omega_0 + \omega_1 LS_i + v_{iJ}, \tag{14a}$$

$$\hat{a}_{iNJ} = \theta_0 + \theta_1 LS_i + v_{iNJ}. \tag{14b}$$

The disturbances in (14) are the estimation errors of the abnormal returns plus errors in approximating the functional form of the relation. Given this, we know the errors are neither independent nor identically distributed (since V^n

is not a scalar matrix and the covariance matrix of the v_i's is proportional to V^n). However, the covariance matrix of the errors can be estimated from the time series residuals from (11).[15] We estimate (14) using ordinary least squares (OLS), weighted least squares (WLS), and generalized least squares (GLS), which correspond to assuming V^n is scalar, diagonal, and block-diagonal, respectively. As before, the blocks are assumed to be defined by three-digit SIC industrial codes. Only the results for the block-diagonal case are presented. These are given in table 8. In addition to the parameter estimates, we show the R^2 value, a sampling-theory-based (Wald) test that the coefficients are jointly zero [see (3.3) on p. 313 of Theil (1971)], and an asymptotic approximation of the posterior odds ratio for the null assuming equal prior odds.

We report the posterior odds ratio since statistical significance at conventional levels (e.g., 0.05 or 0.01) need not imply strong evidence against the null hypothesis in very large samples (we have approximately 1,700 observations in the regressions reported in table 8). This is because when we hold the probability of type I error of the test constant, the probability of type II error goes to zero as the sample size increases. Thus, with large samples all but trivial deviations from the pricing theory will be rejected at conventional significance levels [this property is sometimes referred to as Lindley's paradox; see Zellner (1971, pp. 303–305)]. The asymptotic approximation of the posterior odds ratio, K, for our null hypothesis is a simple function of the F statistic reported in table 8, given by

$$K = \exp\left\{\ln(n-2) - F_{2,n-2}\right\}, \tag{15}$$

where n is the number of observations and $F_{2,n-2}$ is the value of the F statistic [see eq. (54) of Rossi (1980)]. The dependence of the odds ratio on sample size as well as the F statistic is clear from (15).

We first investigate the relation between January-specific abnormal returns and firm size. From a classical sampling-theory point of view the hypothesis that $\omega_0 = \omega_1 = 0$ is rejected (at any significance level above 0.001) in every instance except for the ten-factor APT during 1964–1968 and 1969–1973, and the five-factor APT during 1964–1968 and 1974–1978. In almost every case we find the standard negative relation between $\hat{a}_{iJ}$ and LS_i. However, the posterior odds ratio favors the hypothesis of no size-related January seasonal mispricing in three out of four subperiods for the five-factor APT and in two

[15]Let Z denote the inverse of the cross-product matrix of the regressors in (11). That is, $Z = [(eDF')'(eDF')]^{-1}$. Standard results for the Seemingly Unrelated Regression model [see Zellner (1971, p. 240)] imply that $\mathrm{var}(\hat{a}_J^n - a_J^n)$ and $\mathrm{var}(\hat{a}_{NJ}^n - a_{NJ}^n)$ are proportional to V^n, where the constants of proportionality are the (2,2) and (1,1) elements of Z, respectively. Thus, under the assumption that the true a's are linear functions of the value of LS [i.e., (13)], the error covariance matrices in (14) are proportional to V^n.

Table 8

Cross-sectional regression of estimated January-specific ($\hat{a}_J^n$) and non-January-specific ($\hat{a}_{NJ}^n$) mispricing on logarithm of firm size. Size is defined as market value of equity at the beginning of the subperiod. Regressions are estimated using generalized least squares assuming error-covariance matrix is block-diagonal, where blocks are defined by three-digit SIC codes. Estimates of mispricing and the error-covariance matrix are obtained from time-series regressions of asset excess returns on CRSP equal-weighted index, CRSP value-weighted index, first five factors, and first ten factors. Factors are estimated by asymptotic principal components using 1,487, 1,720, 1,734, and 1,745 securities over the periods 1964–1968, 1969–1973, 1974–1978, and 1979–1983, respectively. Under the null hypothesis of no size-related mispricing, the intercepts and slope coefficients should be equal to zero.

$$\hat{a}_J^n = \omega_0 + \omega_1 LS^n + v_J^n \quad \text{and} \quad \hat{a}_{NJ}^n = \theta_0 + \theta_1 LS^n + v_{NJ}^n$$

	January					Non-January				
	$\hat{\omega}_0$ [a]	$\hat{\omega}_1$ [a]	R^2	Wald test [b] $\omega_0=\omega_1=0$	Odds ratio [c] $\omega_0=\omega_1=0$	$\hat{\theta}_0$ [a]	$\hat{\theta}_1$ [a]	R^2	Wald test [b] $\theta_0=\theta_1=0$	Odds ratio [c] $\theta_0=\theta_1=0$
(A) 1964–1968 (1,447 observations)										
CAPM/Equal-weighted	0.35E-1	−0.33E-2	0.02	14.49	0.001	0.84E-2	−0.90E-3	0.05	37.16	< 0.001
	(4.72)	(−5.15)		(< 0.001)		(5.38)	(−6.62)		(< 0.001)	
CAPM/Value-weighted	0.86E-1	−0.59E-2	0.06	100.29	< 0.001	0.23E-1	−0.16E-2	0.03	142.85	< 0.001
	(11.55)	(−9.63)		(< 0.001)		(14.79)	(−12.80)		(< 0.001)	
APT/Five-factors	0.57E-3	−0.17E-3	< 0.01	0.51	867.72	0.77E-2	−0.71E-3	0.02	16.81	< 0.001
	(0.08)	(−0.28)		(0.60)		(5.19)	(−5.61)		(< 0.001)	
APT/Ten-factors	0.24E-2	−0.53E-3	< 0.01	3.38	49.35	0.69E-2	−0.64E-3	0.02	13.96	0.001
	(0.31)	(−0.77)		(0.03)		(4.76)	(−5.10)		(< 0.001)	
(B) 1969–1973 (1,685 observations)										
CAPM/Equal-weighted	0.12	−0.10E-1	0.09	84.22	< 0.001	−0.20E-1	0.18E-2	0.06	58.82	< 0.001
	(12.35)	(−12.86)		(< 0.001)		(−9.36)	(10.18)		(< 0.001)	
CAPM/Value-weighted	0.18	−0.13E-1	0.13	270.06	< 0.001	−0.33E-1	0.22E-2	0.6	148.12	< 0.001
	(19.45)	(−16.73)		(< 0.001)		(−14.81)	(12.91)		(< 0.001)	
APT/Five-factors	0.14E-1	−0.73E-3	< 0.01	5.05	10.75	−0.31E-2	0.20E-3	< 0.01	2.72	110.64
	(1.51)	(−0.97)		(0.006)		(−1.61)	(1.27)		(0.066)	
APT/Ten-factors	0.14E-2	0.64E-4	< 0.01	0.88	698.53	0.25E-4	−0.15E-4	< 0.01	0.10	1522.65
	(0.15)	(0.08)		(0.42)		(0.01)	(−0.09)		(0.905)	

				(C) 1974–1978 (1,699 observations)						
CAPM/Equal-weighted	0.11	−0.11E-1	0.10	97.97	< 0.001	0.79E-2	−0.68	0.01	11.28	< 0.001
	(11.54)	(−12.90)		(< 0.001)		(4.75)	(−4.67)		(< 0.001)	
CAPM/Value-weighted	0.23	−0.16E-1	0.15	366.42	< 0.001	0.15E-1	−0.10E-2	0.03	52.61	< 0.001
	(21.85)	(−18.38)		(< 0.001)		(8.26)	(−6.94)		(< 0.001)	
APT/Five-factors	−0.23E-2	0.49E-3	< 0.01	1.50	378.65	0.82E-2	−0.70E-3	0.02	14.51	< 0.001
	(−0.23)	(0.58)		(0.22)		(5.31)	(−5.38)		(< 0.001)	
APT/Ten-factors	0.40E-1	−0.38E-2	0.01	10.91	< 0.001	0.59E-2	−0.51E-3	0.01	7.76	0.72
	(3.85)	(−4.30)		(< 0.001)		(3.89)	(−3.94)		(< 0.001)	
				(D) 1979–1983 (1,708 observations)						
CAPM/Equal-weighted	0.54E-1	−0.53E-2	0.05	44.66	< 0.001	0.56E-2	−0.42E-3	0.01	8.63	0.31
	(6.92)	(−8.02)		(< 0.001)		(3.52)	(−3.04)		(< 0.001)	
CAPM/Value-weighted	0.83E-1	−0.63E-2	0.02	61.84	< 0.001	0.11E-1	−0.62E-3	0.06	89.77	< 0.001
	(10.39)	(−9.38)		(< 0.001)		(7.07)	(−4.65)		(< 0.001)	
APT/Five-factors	−0.21E-1	0.10E-2	0.02	16.08	< 0.001	0.87E-2	−0.65E-3	0.02	20.91	< 0.001
	(−2.31)	(1.34)		(< 0.001)		(5.48)	(−4.77)		(< 0.001)	
APT/Ten-factors	−0.85E-2	0.56E-3	0.04	32.65	< 0.001	−0.14E-1	0.68E-3	0.07	8.81	0.26
	(−4.82)	(3.64)		(< 0.001)		(−1.53)	(0.84)		(< 0.001)	

[a] T statistics in parentheses.
[b] p-values for Wald test which has an F distribution with degrees of freedom of 2 and (number of observations -2) under the null hypothesis.
[c] Asymptotic approximation of the posterior odds ratio (odds in favor of null/odds in favor of alternative), assuming equal prior odds.

subperiods for the ten-factor APT. This is consistent with the results for the size-grouped portfolios shown in fig. 2.

The results for non-January-specific returns are also consistent with those reported above for the size-grouped portfolios. We find a reversal in the size effect in the 1969–1973 subperiod. For the CAPM there is a *positive* relation between our measures of abnormal returns and firm size. During the other periods there is the usual negative relation between abnormal returns and size. This is consistent with the findings of Brown, Kleidon, and Marsh (1983), which show a reversal of the size effect in the 1969–1973 period. The posterior odds ratio for $\theta_0 = \theta_1 = 0$ favors the null hypothesis only in the 1969–1973 period. These results are consistent with those depicted in fig. 3.

Our final test involves a restriction implied by an intertemporal version of the equilibrium APT [see Connor and Korajczyk (1987)]. This model implies there is a factor (say, the first factor) for which each asset has a sensitivity of unity. That is, $B^n = (e^n \beta^n)$, where e^n is an n-vector of ones. We call this factor the unit-beta factor. If we could observe the true factors, F, and if we knew which factor corresponded to the unit-beta factor, then we could easily test the linear restriction

$$B^n_{.1} = e^n, \tag{16}$$

where $B_{.i}$ represents the ith column of B and we have assumed that the factors have been ordered so that the first factor is the unit-beta factor. However, we actually observe $G^n = L^n F + \phi^n$. Even if we assume $\phi^n \approx 0$ so $G^n \approx L^n F$, there is a rotational indeterminacy problem that prevents us from testing (16) directly. From (4) and assuming $G^n = L^n F$, we have that

$$R^n = \left[B^n (L^n)^{-1}\right] G^n + \varepsilon^n = B^{n*} G^n + \varepsilon^n.$$

Since L^n is unobservable, the only restriction imposed on B^{n*} is that there exists a k-vector λ^n (corresponding to the first column of L^n) such that

$$B^{n*} \lambda^n = e^n. \tag{17}$$

This is a nonlinear (since λ^n is unknown) restriction on the coefficient matrix B^{n*}. Let us partition B^{n*} into B^{n*}_1, a $k \times k$ matrix formed from the first k rows of B^{n*}, and B^{n*}_2, a $(n-k) \times k$ matrix formed from the remaining rows of B^{n*} (to simplify notation we will drop the n superscript except when this might cause some confusion). Thus, $B^{*\prime} = (B^{*\prime}_1, B^{*\prime}_2)$. The restriction (17) implies that

$$B^*_1 \lambda = e^k \quad \text{and} \quad B^*_2 \lambda = e^{n-k}. \tag{18}$$

Table 9

Wald test for unit-beta restriction on five-factor APT for ten portfolios (equal-weighted portfolios based on a ranking of market value at the beginning of each five-year subperiod). The restriction implies that there is a linear combination of the five factors such that each of the portfolios has a sensitivity, with respect to the linear combination, equal to 1.0. Asymptotically, the statistic has F distribution with degrees of freedom $\nu_1 = 5$ and $\nu_2 = 540$ under the null hypothesis.

Period	F statistic	(p-value)
1964–1968	6.62	(0.00)
1969–1973	6.27	(0.00)
1974–1978	2.36	(0.04)
1979–1983	323.76	(0.00)

The first equality in (18) implies that $\lambda = (B_1^*)^{-1}e^k$. Inserting this into the second equality in (18) we get the following nonlinear cross-equation restrictions on the parameters of the model:

$$B_2^*(B_1^*)^{-1}e^k - e^{n-k} = \mathbf{0}. \tag{19}$$

We use a Wald test [see Gallant (1987, p. 328)] to test (19). The results for the five-factor APT using size-based portfolios are reported in table 9. (The tests are not feasible using disaggregated data and there are no overidentifying restrictions for the ten-factor model.) The tests reject the unit-beta restriction, at the 0.05 level, for each subperiod. The rejections are quite strong except in the 1974–1978 subperiod (which has a p-value of 0.04). It is difficult to determine which aspects of the intertemporal model are leading to the rejections in table 9. One possibility is that the option-like features of common stock, caused by risky debt in the capital structure, create nonlinearities in the factor structure that may invalidate the pricing restrictions [Jagannathan and Korajczyk (1986) discuss problems caused by nonlinearities in a CAPM context]. It may be that the tests of the unit-beta restriction have more power against this alternative than do tests of the intercept restriction. See Connor and Korajczyk (1987) for a more detailed discussion of the intertemporal equilibrium version of the APT.

5. Summary

This paper implements a new set of econometric techniques for estimating and testing the APT, using the asymptotic principal components theory first suggested by Chamberlain and Rothschild (1983) and extended by Connor and Korajczyk (1986).

Section 3 extends the asymptotic principal components technique further. We develop a more efficient version of the estimator, and we show that the techniques are valid for some cases of time-varying risk premiums.

In section 4 we test the APT and CAPM using both size-grouped portfolios and large numbers of individual assets. The tests with individual assets are made possible by placing prior restrictions on the structure of the covariance matrix of idiosyncratic returns.

For mispricing that is not January-specific our five-factor version of the APT seems to perform better than the value-weighted CAPM and about as well as the equal-weighted CAPM. The APT performs much better than either implementation of the CAPM in explaining the January-specific mispricing related to firm size. This result is due to seasonality in the estimated risk premiums of the multi-factor model that is not captured by the single-factor CAPM relations, even though the premium in the latter model also exhibits seasonality.

We also test the prediction of an intertemporal version of the APT that there is a factor for which all assets have a sensitivity of unity. This hypothesis is strongly rejected for a five-factor APT.

Extensions of this work can take several directions. Procedures designed to compare nonnested models [similar to those used in Chen (1983)] will improve our understanding of the relative merits of the models. Some improvement in the technology may be obtained by investigating different specifications of the error covariance matrix, V^n. Linking the seasonality in estimated factor risk premiums to more fundamental economic variables should help us understand the nature of the observed seasonal effects.

Our empirical results indicate that while neither of our implementations of the APT or CAPM is a perfect model of asset pricing, the APT is consistent with the persistent size-related seasonal effects in asset pricing. Empirically, the model seems to be a reasonable alternative to the CAPM.

References

Berndt, Ernst R. and N. Eugene Savin, 1977, Conflict among criteria for testing hypotheses in the multivariate linear regression model, Econometrica 45, 1263–1277.

Binder, John J., 1985, On the use of the multivariate regression model in event studies, Journal of Accounting Research 23, 370–383.

Black, Fischer, Michael C. Jensen, and Myron Scholes, 1972, The capital asset pricing model: Some empirical tests, in: Michael C. Jensen, ed., Studies in the theory of capital markets (Praeger, New York, NY) 79–121.

Brown, Philip, Allan W. Kleidon, and Terry A. Marsh, 1983, New evidence on the nature of size-related anomalies in stock prices, Journal of Financial Economics 12, 33–56.

Chamberlain, Gary and Michael Rothschild, 1983, Arbitrage and mean variance analysis on large asset markets, Econometrica 51, 1281–1304.

Chan, K.C., Nai-fu Chen, and David A. Hsieh, 1985, An exploratory investigation of the firm size effect, Journal of Financial Economics 14, 451–471.

Chen, Nai-fu, 1983, Some empirical tests of the theory of arbitrage pricing, Journal of Finance 38, 1393–1414.

Chen, Nai-fu, Richard Roll, and Stephen A. Ross, 1986, Economic forces and the stock market, Journal of Business 59, 383–403.

Connor, Gregory, 1984, A unified beta pricing theory, Journal of Economic Theory 34, 13–31.

Connor, Gregory and Robert A. Korajczyk, 1986, Performance measurement with the arbitrage pricing theory: A new framework for analysis, Journal of Financial Economics 15, 373–394.

Connor, Gregory and Robert A. Korajczyk, 1987, An intertemporal equilibrium beta pricing model, Working paper no. 37 (Department of Finance, Northwestern University, Evanston, IL).

Fama, Eugene F. and James D. MacBeth, 1973, Risk return, and equilibrium: Empirical tests, Journal of Political Economy 71, 607–636.

Ferson, Wayne E., Shmuel Kandel, and Robert F. Stambaugh, 1987, Tests of asset pricing with time-varying expected risk premiums and market betas, Journal of Finance 42, 201–220.

Gallant, A. Ronald, 1987, Nonlinear statistical models (Wiley, New York, NY).

Gibbons, Michael, 1982, Multivariate tests of financial models: A new approach, Journal of Financial Economics 10, 3–28.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1986, A test of the efficiency of a given portfolio, Research paper no. 853 (Graduate School of Business, Stanford University, Stanford, CA).

Huberman, Gur and Shmuel Kandel, 1985, A size based stock returns model, Working paper no. 148 (Center for Research in Security Prices, University of Chicago, Chicago, IL).

Ibbotson Associates, 1985, Stocks, bonds, bills, and inflation, 1985 Yearbook.

Ibbotson, Roger C., 1979, The corporate bond market: Structure and returns, Unpublished manuscript (Graduate School of Business, University of Chicago, Chicago, IL).

Jagannathan, Ravi and Robert A. Korajczyk, 1986, Assessing the market timing performance of managed portfolios, Journal of Business 59, 217–235.

Jensen, Michael C., 1968, The performance of mutual funds in the period 1945–1964, Journal of Finance 23, 389–416.

Jobson, J.D., 1982, A multivariate linear regression test for the arbitrage pricing theory, Journal of Finance 37, 1037–1042.

Keim, Donald B., 1983, Size-related anomalies and stock return seasonality: Further empirical evidence, Journal of Financial Economics 12, 13–32.

Keim, Donald B., 1985, Dividend yields and stock returns: Implications of abnormal January returns, Journal of Financial Economics 14, 473–489.

Keim, Donald B. and Robert F. Stambaugh, 1986, Predicting returns in the stock and bond markets, Journal of Financial Economics 17, 357–390.

Lehmann, Bruce N. and David M. Modest, 1988, The empirical foundations of the arbitrage pricing theory, Journal of Financial Economics, this issue.

Morrison, Donald F., 1976, Multivariate statistical methods, 2nd ed. (McGraw-Hill, New York, NY).

Muirhead, Robb J., 1982, Aspects of multivariate statistical theory (Wiley, New York, NY).

Rao, C.R., 1973, Linear statistical inference and its applications, 2nd ed. (Wiley, New York, NY).

Ross, Stephen A., 1976, The arbitrage theory of capital asset pricing, Journal of Economic Theory 13, 341–360.

Rossi, Peter E., 1980, Testing hypotheses in multivariate regression: Bayes vs. non-Bayes procedures, Unpublished manuscript (Graduate School of Business, University of Chicago, Chicago, IL).

Shanken, Jay, 1985a, Multivariate tests of the zero-beta CAPM, Journal of Financial Economics 14, 327–348.

Shanken, Jay, 1985b, Multi-beta CAPM or equilibrium APT? A reply, Journal of Finance 40, 1189–1196.

Stambaugh, Robert F., 1982, On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis, Journal of Financial Economics 10, 237–268.

Theil, Henri, 1971, Principles of econometrics (Wiley, New York, NY).

Zellner, Arnold, 1971, An introduction to Bayesian inference in econometrics (Wiley, New York, NY).