THE LOAN CONTRACT: MECHANISM OF FINANCIAL CONTROL*

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Industry banking, the style of banking under which a single bank satisfies all the external financial needs of an industry, was recognised by Alfred Marshall (1919) as a positive force in the development of new industries. Today many of Marshall's reference industries have reached maturity but their industry banks continue to play an active role not only in financing their production and expansion, but also in their management (Cable, 1985; Prais, 1981). To understand the economic impact of industry banking, this paper explores one aspect of it: bank directed industry coordination and control.

Although industry banking is acknowledged as a viable style of banking (Cable, 1985; Kotz, 1978), it has been virtually ignored by banking theorists as a framework for the analysis of bank behaviour. Two other perspectives dominate the banking literature; the first views a bank as a joint product firm, the second views a bank as a Tobinesque portfolio allocator (Baltensperger, 1980). Neither of these perspectives takes a bank's power as a debt contractor into account. This oversight is remedied in Stiglitz and Weiss (1981) and Hester and Pierce (1975), who both analyse ways in which a bank can affect the distribution of returns on its loans by altering the ways in which they are written. They both emphasise the importance to banks of information about their borrowers, but neither consider the possibility of using information about one borrower when writing a loan contract for another. Stiglitz (1985) takes the analysis one step further in his discussion of how a debt contract can be designed to constrain a firm's behaviour effectively, but he does not extend his analysis to the case of industry control.

Industry control is based on a bank's ability to compile information that is in general not available to individual firms, and then use that information to write behaviour modifying loan contracts. Section II presents a model in which a bank writes incentive compatible contracts that elicit and disseminate firm specific information which is important to firm decision-making but is not revealed by the market. The contracts, which hold the firms to the multiplant monopoly level of output, are not breached because of severe penalties specified in negative covenants similar to those found in real loan contracts. If certain feasibility conditions are satisfied, industry-coordinating contracts can be written whether or not firms have internal funds, and will be written so long as it is profitable for the bank. The contracts are similar to the principal-agent

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contracts developed in Demski and Sappington (1984), while the information-sharing process closely resembles Gerard-Varet and Moulin's (1978) correlating mechanism.

To provide the theoretical analysis in Section II with an institutional base, Section I discusses the economic forces which may lead to a banking industry segmented into industry banks. Section III concludes the paper with a brief discussion of extensions to and the testable implications of the analysis.

I. COST OF FINANCING PRODUCTION AND INDUSTRY BANKING

A firm seeking outside funding for production or expansion can either raise money directly on the capital market by issuing equity or debt, or it can take out a bank loan. The first method requires the services of an investment banker, the second the services of a commercial banker. Both types of banks require a firm to disclose detailed information about itself, and so firms needing outside funds both today and in the future might choose to use repeatedly the financial services of a particular commercial or investment bank to reduce the costs of disclosure. Since investment banks charge a fixed, per-issue underwriting fee, the average cost of a small issue is high, which may close small or new firms out of the capital market and force them into borrowing from commercial banks. Large firms might also avoid issuing equity to raise funds, since in widely held firms the controlling group might be altered (Cubbin and Leech, 1983; Leech, 1984), or the probability of a successful takeover bid might be increased.

If firms show a preference for commercial over investment banks, and if the returns to specialisation exceed those to diversification, it might be in a bank's interest to become an expert in lending to a particular industry. Having established itself as an industry expert, a bank may see little competition if the fixed cost of gaining expertise serves as a barrier to entry. New and existing firms in the industry would go, or return, to the industry bank because of its expertise, and other banks would expect loan requests to them from that industry to be the expert bank's rejects. Under these circumstances the industry bank is in the position to exert control.

II. LENDING TO AN INDUSTRY WITH STOCHASTIC TECHNOLOGIES

II.1. The Model

The model represents a contractual relationship between a bank and a duopoly. Three situations are analysed: in the first the firms do not have any internal funds and must borrow to finance production, in the second the firms have internal funds about which the bank is fully informed, and in the third the firms have internal funds about which the bank is uninformed. To concentrate on the bank’s role as an information broker, assume that all agents can borrow and lend without condition from the capital market at an interest rate $r$.

The duopolists each maximise expected profits over a single period. A firm's
output, $y$, is a random function of funding, $F$, $y^j_n = \theta^j_n q(F)$, where $\theta^j_n \geq 0$ is a productivity scaler for firm $j$ in state $n$, and $q'(F) > 0$ for all $F > 0$, $q(o) = 0$.

To produce output $y$ a firm incurs two types of costs: those relating directly to production, $C(y)$, and those relating to financing, $rF$. A firm $j$ decides how much to produce given its information set which includes

(i) the market interest rate, $r$,
(ii) the industry inverse demand function, $p(y^1, y^2)$,
(iii) its productivity $\theta^j_n$, a realisation of the random variable $\tilde{\Theta} = \{\theta^j_n\}_{n=1}^N$, which occurs with probability

$$\zeta_n \sum_{n=1}^N \zeta_n = 1,$$

and (iv) its unbiased expectations of the other firm's productivity,

$$\sum_{m=1}^N \zeta_m (\theta^k_m),$$

where $\theta^j_n$ and $\theta^k_m$ ($j, k = 1, 2; n, m = 1, \ldots, N$) are independent for all $n, m, j \neq k$.

Assume the firms form conjectures about each other's output, and act in such a way that an equilibrium exists. If the firms' only source of external funds is the capital market, and they have no internal funds, in equilibrium firm $j$'s expected profits are

$$E[\pi^j(\tilde{\Theta})] \quad (j = 1, 2).$$

Although the duopolists would earn the highest expected profits in the collusive equilibrium, they will not collude since neither firm has the incentive to reveal truthfully its productivity state to the other firm. For example, suppose firm 1 tells the truth; it cannot force firm 2 to reciprocate and also tell the truth; while firm 2 cannot be confident that firm 1 has not lied. If the firms could make binding commitments by posting bonds, they could guarantee that their signals were accurate, but such an action is tantamount to explicit collusion which is in violation of antitrust statutes.

Banks provide the duopolists with an alternative means of financing their production and a possible means of colluding. If the duopolists choose to borrow from banks the only interesting case is that in which they both borrow from the same one, since competing banks cannot improve on the market solution without colluding. So, suppose both firms decide to borrow from the same bank – an expected profit-maximising agent with net worth and enough a priori information to carry out the two-plant monopoly expected profit maximisation programme.\(^1\) (The bank knows, at least, the distribution of both firms' productivity scalers, the base production and cost functions, and the industry demand function.) If the bank can write state contingent loan contracts that are acceptable to the firms and that force the firms to produce at the joint maximising level, it can induce collusive behaviour.

\(^1\) The assumption that this information is freely available in the market is not necessary. All that is required is that the bank has already undertaken the costly process of gathering the information. After the information is collected, the cost is sunk and so may be ignored in the bank's optimisation programme.
To determine contractual terms when the firms do not have any internal funds, the bank solves

$$\text{maximise} \sum_{n=1}^{N} \sum_{m=1}^{N} \xi^1_n \xi^2_m \left( \rho \left[ y^1(\theta^1_n | \theta^2_m), y^2(\theta^2_m | \theta^1_n) \right] + y^1(\theta^1_n | \theta^2_m) + y^2(\theta^2_m | \theta^1_n) \right)$$

$$- \left\{ C \left[ y^1(\theta^1_n | \theta^2_m) \right] + C \left[ y^2(\theta^2_m | \theta^1_n) \right] \right\}$$

$$- r \left( q^{-1} \left[ y^1(\theta^1_n | \theta^2_m) \right] + q^{-1} \left[ y^2(\theta^2_m | \theta^1_n) \right] \right) \right\} \right\}$$

subject to

$$E \left[ \pi^*(\hat{\theta}^1, \hat{\theta}^2) \right] \geq E \left[ \pi^1(\hat{\theta}^1) + \pi^2(\hat{\theta}^2) \right], \quad (3)$$

$$\hat{\theta}^1(\theta^1_n) \geq E \left[ \pi^1(\theta^1_n) \right] \quad (j = 1, 2; n = 1, \ldots, N), \quad (4)$$

$$\hat{\theta}^2(\theta^2_m) \geq E \left[ \pi^2(\theta^2_m) \right] \quad (j = 1, 2; n = 1, \ldots, N), \quad (5)$$

$$\hat{\pi}^l(\theta^1_n) \geq (\hat{\pi}^l(\theta^1_n) + \max \{ \Delta q^{-1} \left[ y(\theta^1_n) \right] \} ) \quad (n, l = 1, \ldots, N), \quad (6)$$

where \( r(\cdot) \) is the bank's cost of funds, and \( q^{-1}(y) \) is the level of funding required to produce output \( y \).

In solving (2) the bank determines the optimal output pairs for all contingencies. For example, \( y^1(\theta^1_n | \theta^2_m) \) is firm 1's optimal output when its productivity is \( \theta^1_n \), and firm 2's productivity is \( \theta^2_m \). This joint state occurs with probability \( \xi^1_n \xi^2_m \). Having determined the output levels, the bank must write contracts to make this state contingent production scheme possible and profitable. For the contracts to be profitable for the bank they must satisfy constraint (3) by ensuring that expected industry profits, \( E \left[ \pi^*(\hat{\theta}^1, \hat{\theta}^2) \right] \), exceed the sum of the expected contractually determined profits \( E \left[ \pi^1(\hat{\theta}^1) + \pi^2(\hat{\theta}^2) \right] \), where the \( ' \wedge ' \) denotes a contractually set value. To satisfy constraint (4), a contract must be acceptable to a firm so state contingent contractual profits, \( \hat{\pi}^l(\theta^1_n) \), must be at least as large as expected duopoly profits for that state, \( E \left[ \pi^l(\theta^1_n) \right] \). Since a firm only knows its own productivity state, contractual profits for a firm must be contingent on its productivity state alone.

To guarantee that the correct production plan for the joint state of nature is put into effect, the bank must provide incentives so each firm's dominant strategy is to reveal truthfully its productivity state. To ensure that a firm produces at the level consistent with its loan and its announced productivity state, the bank prohibits under- or over-production and, implicitly, capital market activity, (5). If a firm claims to be in a high productivity state when it is in a low one, it will not be able to produce adequate output. Underproducing or borrowing to meet its production goal reveals its misrepresentation of its productivity and can be punished. If a firm claims to be in a low productivity state when it is in a high one, it could over-produce or lend out the part of the loan not required to meet its production goal, but since these actions reveal misrepresentation they can be punished. To ensure that firms abide by constraint (5), covenants of the contract must specify reported state contingent penalties, \( b(\theta^1_n) \), for under- or over-production or capital market activity. These penalties take the form of negative covenants in real loan contracts. 2 Finally,
by constraint (6), a firm’s contractual profits if it claims productivity \( \theta_n \), \( \hat{\theta}_n(\theta_n) \), must exceed the maximum it could get if its productivity state were \( \theta_i, \hat{\theta}_i(\theta_i) \), and it keeps the part of the loan not required to produce the contractually set level of output, \( \max\{q^{-1}(y(\theta_i^2 | \theta^k_m)), m = 1, \ldots, N\} \) (I = 1, \ldots, N).^{3}

The contract offered by the bank can now be defined:

**Definition 1.** A feasible contract is a state-contingent profit level and a loan covenant, \([\hat{\theta}_n(\theta_n); b^i(\theta_n)]\), that satisfies constraints (3)–(6) for all \( j = 1, 2, n = 1, \ldots, N \).

When a firm accepts a contract, it reveals its productivity state to the bank and not to the other firm. Using this information from both firms, the bank acts like Gerard-Varet and Moulin’s (1978) correlating mechanism by signalling to each firm its optimal action; however, unlike the correlating mechanism the bank exacts a fee for its services. It does this by setting loans, which tell the firms how much to produce, and interest rates, which determine how profits are redistributed.

The question remains whether the bank can write contracts that solve (2) and are feasible. Proposition 1 shows that if feasibility can be established, an optimal contracts exists.

**Proposition 1.** Suppose there are only two productivity states possible for each duopolist (\( N = 2 \)). Then, if a feasible (definition 1) contract exists, the bank can offer each duopolist a contract that

(i) constrains the duopolist to reservation (expected duopoly) profits, in the low-productivity state,\(^4\)

(ii) constrains output to the collusive (full-information, efficient (joint profit maximising)) level.

**Proof.** See Appendix.

The contracts specify the loan amounts that allow the collusive level of output to be produced, the loan covenants that ensure these output targets are exactly met, and the interest rates that achieve the agreed upon profit sharing.\(^5\) The sequence of events is shown in Table 1.

Proposition 1 holds only if firms do not have any internal funds with which to finance production. If firms do have internal funds, the bank’s problem must be altered to take this into account. Suppose each firm begins period 1 with internal funds \( F_j, (j = 1, 2) \) and these levels are market information. The bank’s problem is unaltered, however, under constraint (5), although firms are still explicitly prohibited from under- or over-producing, and implicitly prohibited from borrowing, they are now implicitly required to lend out all their internal funds. Penalties, \( b^J(\theta_n) \), are set to ensure that the explicit and implicit conditions of the constraints are not violated. The profit constraint, (6), is not altered either, since the firm’s expected duopoly profits are invariant to the source of non-bank funds. (The opportunity cost of a dollar of funding

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3 The general problem of eliciting truthful revelation of inside information is found in many economic applications. For example, a similar problem is faced by Meyerson’s (1979) arbitrator.

4 The reservation level of profits is that attained by a duopolist under any set of conjectures that leads to an equilibrium. When the bank sets the contract it knows the conjectural method used by the duopolists.

5 These contracts are very similar to those derived by Demski and Sappington (1984) in a principal-agent model. Their contracts prevent shirking; these enforce collusion.
is $r$ whether the firm borrows it from the capital market or uses its own funds.) In this case, also, if a feasible contract exists, an optimal contract can be written.

**Definition 2.** A feasible contract is a state contingent profit level and loan covenant, $[\hat{\pi}(\theta^l_n); b^{\ell}(\theta^l_n)]$, that satisfies constraints (3)–(6), for all $j = 1, 2$, $n = 1, \ldots, N$.

**Proposition 2.** If the duopolists have internal funds the levels of which are market information, and $N = 2$, then, if a feasible (Definition 2) contract exists, the bank can offer each duopolist a contract that

(i) constrains the duopolist to reservation profits in the low productivity state,

(ii) constrains output to the collusive level.\(^6\)

If firms have internal funds, the level of which is private information, the bank may still be able to write industry-controlling contracts. In this case constraint (5) is again unaltered, but although over- and under-production are explicitly prohibited, only borrowing is implicitly prohibited. Penalties, $b^{\ell}(\theta^l_n)$, are imposed so that constraint (5) is not violated. Constraint (6) is revised to take into account the bank’s inability to distinguish sources of funds

$$\hat{\pi}(\theta^l_n) \geq \hat{\pi}(\theta^l_i) + r \max \{\Delta q^{-1}[y(\theta^l_i|\theta^m_n)], m = 1, \ldots, N\} \quad (n, l = 1, \ldots, N). \quad (6')$$

**Definition 3.** A feasible contract is a state contingent profit level and loan covenant, $[\hat{\pi}(\theta^l_n); b^{\ell}(\theta^l_n)]$, that satisfies constraints (3)–(5), and (6'), for all $j = 1, 2$, $n = 1, \ldots, N$.

**Proposition 3.** Suppose that both duopolists have internal funds the levels of which are private information, and $N = 2$. Then, if a feasible (Definition 3) contract exists, the bank can offer each duopolist a contract that

(i) constrains the duopolist to reservation profits in the low-productivity state,

(ii) constrains output to the collusive level.\(^7\)

\(^6\) Since the opportunity cost of funds to the firm is $r$ whether it uses its own funds or borrows from the market, the proof of Proposition 2 is identical to the proof of Proposition 1.

\(^7\) The method of proof for Proposition 3 is the same as that for Proposition 1 and is available upon request from the author.
In this and the preceding case the bank, essentially, agrees to borrow the firms’ internal funds at the market rate, and then lends all or part of those and possibly other funds to the firms so that they can produce at the collusive level.

II.2. Discussion

In Propositions 1–3 contract feasibility depends largely on whether coordinating the collusive arrangement is expected to be profitable for the bank. Profitability depends, in part, on whether the bank has adequate net worth to meet the contractual agreement in those states in which joint maximising profits are insufficient.8 All else equal, feasibility is least likely if firms have internal funds about which the bank is uninformed since the costs of maintaining the relationship may be higher, and the conditions under which the bank can induce the firms to tell the truth are more restrictive. However, the close working relationships between firms and their industry bank make the possibility that the bank is totally uninformed about the firms’ internal funds seem very unlikely.

The firms in this model are observationally equivalent from the perspective of the market, and so any loan contract which differentiates between them suggests that credit is being rationed.9 However, using its superior information the bank is able to distinguish between the firms and to write state contingent contracts which allocate capital efficiently; this is only in a limited sense credit-rationing.

In the contractual arrangements, as described in the proof to Proposition 1, if a feasible contract exists the bank writes the optimal contract to maximise its profits from control by setting firm profit levels at the minimum consistent with truth-telling and individual rationality. If feasibility conditions have been satisfied, an optimal contract can be written under which the surplus control is, for example, shared proportionally. The bank enforces all types of controlling contracts in the same fashion by first collecting and then redistributing the surplus.

If bank-directed industry control is feasible, collusion-enforcing contractual agreements are possible in banking systems with multiple banks. Suppose there are two banks in the system and firm 1 borrows from bank 1 and firm 2 borrows from bank 2, then there is no reason for the banks to exist since they cannot improve on the market. However, if the firms come as a package the banks

8 The negative carry is the reason a bank requires net worth for all contracts to be feasible. In Holmstrom (1982) a similar net worth requirement is mandated for incentive-compatible contracts to be feasible. Though in a single-period framework one would expect the bank to renege, I rule out this possibility.

9 In this model a bank requires a lot of information to be able to write a collusion-inducing surplus-extracting contract. In much of the recent literature on credit-market activity such as Stiglitz and Weiss (1981), banks have been assumed to be unable to gather such detailed information. These models best apply to consumer lending markets in which loan size is small and credit evaluations which could distinguish good and bad risks are too costly to be justified. In the model in this paper loan size is large, and so costly credit evaluation is merited. Obviously, if there were costs of extracting the information from the firms, it is possible that a bank would choose to stop well short of the perfect information it is costlessly able to elicit in this model. However, so long as a bank’s information is better than each individual firm’s, it may be able to write a surplus extracting contract.
could divide any surplus derived from control, by, for example, a linear sharing rule. This sort of arrangement is analogous to a consortium loan in which the lead bank administers the loan and each of the participants share in the profits.

Propositions 1–3 can be generalised for \( N > 2, J > 2, N, J \) finite. Including explicit financing costs (costs of information collection and evaluation for the bank, and costs of disclosure for the firms) in the model seems straightforward. Intuitively, these extensions are possible since loan contracts between a bank and each firm can be made arbitrarily elaborate.

If the duopolists were risk-averse rather than risk-neutral, in many cases controlling-contracts could still be written. The contract would be very similar in form to the fixed wage contract of implicit contracting theory (Azariadis, 1975) in that it would offer a duopolist an own-state component contingent fixed-profit contract under which the profit share would be less than that required under risk neutrality. Risk aversion on the part of the bank, however, could complicate matters greatly and introduce the possibility that a feasible contract would not exist.

The most important assumptions of the model are that each firm receives, prior to taking its production decision, a perfect own-productivity signal that is totally uninformative about its rival’s productivity, and that neither can signal its productivity because of incentive problems. This inability of firms to communicate offers a bank the possibility of effecting industry control. A trade association as well as a bank may be capable of being an information conduit between firms. However, the imposition of penalties by a trade association on member firms for non-cooperative behaviour is more likely to be found in violation of antitrust statutes than similar behaviour by banks.

III. CONCLUSION

The model just discussed has illustrated how, in an economy characterised by asymmetric information, a bank through its loan contracts can elicit, evaluate, and disseminate information, affect industry output, and provide a mechanism by which economic surplus is distributed. The well-constructed debt contract rather than the equity share provides the bank with its ability to control. Other

10 This could be modelled as a super game. If the colluding banks play a trigger strategy and if the discount rate is ‘right’, collusion will always be the dominant strategy.
11 Shavell (1984) points out that if a state of nature is very unlikely, conditions restricting actions in that state may not be included in a contract.
12 A properly modified constraint (3) could be incompatible with constraints (4)–(6).
13 The passage of time may undermine a bank’s influence. If so, industry banks would function in multiple capacities by exerting a controlling influence over some industries and by acting as a competitive banker in others. From the perspective of an external observer it would be difficult to determine in which industry it played which role.
14 In U.S. antitrust case-history there has been only one case in which a bank has been named as a member of a collusive arrangement among non-financial firms (Kotz, 1978). This is because loan contracts and other financial contracts do not establish any relationship between participating firms. Such a claim is more difficult to substantiate in contractual arrangements between a trade association and its member firms.
15 The model presented applies to a single-period loan. Pecchenino (1985) presents a multiperiod model of bank coordination of industry based on the limit-pricing model developed in Kamien and Schwartz (1971).
market imperfections may also create the potential for bank-directed collusion. For example, if information is perfect but capital markets are not, because banks can borrow more cheaply than firms, a similar contracting relationship between banks and firms can be established. For a specific model in which the industry is a symmetric duopoly with deterministic costs, it can be shown that if a feasible contract (one which does not violate either firm or bank individual rationality) exists, an optimal contract under which output is held to the industry profit-maximising level and the surplus is divided among the firms and the bank can be written. Further, if the bank is constrained by the government to make zero profits, an output-maximising contract, if feasible, exists.

Whether banks actually do control industries is an empirical question. Ideally the determination of bank control would be revealed by a bank’s loan contracts. If, as is suggested by the model, the contracts extended to firms in the same industry specify negative covenants which prohibit outside borrowing, many uses of internal funds, and the rate at which capital is worn out, and if loan interest rates vary positively with firm net revenues prior to loan repayment, then industry control may be suspected. Since loan contracts are private documents, more indirect indicators of bank coordination might necessarily be relied upon. For example, bank-directed industry control might be suspected if most firms that are in the same industry and operating in the same bank borrow from the same bank, or if individual (controlled) firm output in an industry producing good x in one market is more variable than individual firm output in an industry producing good x in another similar market. Another possible indicator is bank officials sitting on the boards of many firms’ in the same industry, or being present at executive committee meetings of these same firms. None of these indicators prove bank control of industry, but rather suggest it as a possibility.

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APPENDIX

Proof of Proposition 1

Suppose the productivity states are ordered so $\theta_1 < \theta_2^j (j = 1, 2)$. Consider the following contract for duopolist 1:

If the firm’s productivity state is low, $\theta = \theta_1^1: [\hat{\pi}(\theta_1^1); b(\theta_1^1)]$.

Suppose there are two duopolies, each of which produce good x in different local markets, each of which is characterised by the same market inverse demand function. Assume duopoly 1 is controlled by a bank while duopoly 2 is not, each firm in each industry is either in a high or a low productivity state with equal probability, and the firms’ shared production technology is constant returns to scale. In duopoly 2 both firms always produce. However, in duopoly 1, when one firm is in its high productivity state and the other firm is in its low productivity state, the productive firm produces the entire industry output while the unproductive firm remains idle.

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16 This model has been developed and is available from the author upon request.

17 Suppose there are two duopolies, each of which produce good x in different local markets, each of which is characterised by the same market inverse demand function. Assume duopoly 1 is controlled by a bank while duopoly 2 is not, each firm in each industry is either in a high or a low productivity state with equal probability, and the firms’ shared production technology is constant returns to scale. In duopoly 2 both firms always produce. However, in duopoly 1, when one firm is in its high productivity state and the other firm is in its low productivity state, the productive firm produces the entire industry output while the unproductive firm remains idle.
If the firm’s productivity state is high, $\theta = \theta^2_2$, $b(\theta^1_2)$, where

$$b(\theta^1_1) = \max \{ p[y^1(\theta^1_1 | \theta^2_1), y^2(\theta^2_1 | \theta^2_2)] y^1(\theta^2_1 | \theta^2_2) - C[y^1(\theta^1_1 | \theta^2_2)] + r \Delta q^{-1}[y^1(\theta^2_1 | \theta^2_1)] + p[y^1(\theta^1_1 | \theta^2_1), y^2(\theta^2_1 | \theta^2_2)] y^1(\theta^2_1 | \theta^2_2) - C[y^1(\theta^2_1 | \theta^2_2)] + r \Delta q^{-1}[y^1(\theta^2_1 | \theta^2_2)] \},$$

$$b(\theta^2_2) = \max \{ p[y^1(\theta^1_1 | \theta^2_1), y^2(\theta^2_1 | \theta^2_1)] y^1(\theta^1_1 | \theta^2_2) - C[y^1(\theta^1_1 | \theta^2_2)] + r \Delta q^{-1}[y^1(\theta^2_1 | \theta^2_1)] + p[y^1(\theta^1_1 | \theta^2_1), y^2(\theta^2_1 | \theta^2_1)] y^1(\theta^1_1 | \theta^2_2) - C[y^1(\theta^2_1 | \theta^2_2)] + r \Delta q^{-1}[y^1(\theta^2_1 | \theta^2_2)] \},$$

$$\hat{\pi}(\theta^1_1) = E[\pi(\theta^1_1)],$$

$$\hat{\pi}(\theta^2_2) = \max (E[\pi(\theta^2_2)]; \hat{\pi}(\theta^1_1) + \max \{ \Delta q^{-1}[y^1(\theta^1_1 | \theta^2_2)], \Delta q^{-1}[y^1(\theta^1_1 | \theta^2_2)] \}),$$

and where $\Delta q^{-1}$ is the difference between the loan extended given the firm’s claimed productivity state and the loan required to produce the contracted level of output in the firm’s actual productivity state.

For the duopolist to accept the contract and truthfully reveal its productivity state, the following conditions must be satisfied:

$$IR \quad \hat{\pi}(\theta^1_n) \geq E[\pi^1(\theta^1_n)] \quad (n = 1, 2),$$

$$TT1 \quad \hat{y}(\theta^1_n) = y(\theta^1_n) \quad (n = 1, 2),$$

$$TT2 \quad \hat{\pi}(\theta^1_n) \geq \hat{\pi}(\theta^1_n) + \max \{ \Delta q^{-1}[y^1(\theta^1_1 | \theta^2_m)], m = 1, 2 \} \quad (l, n = 1, 2).$$

The individual rationality (IR) and truth-telling (TT1 and TT2) constraints are equivalent to constraints (4)–(6) for firm 1 in problem (2). The penalties are set so that misrepresenting one’s productivity state is unprofitable.

$b(\theta^1_1)[b(\theta^2_1)]$ is the penalty imposed if duopolist 1 claims to be in his low (high) productivity state but is revealed to be in his high (low) productivity state. The penalties are equal to the maximum profits he could make by these misrepresentations regardless of the other firm’s productivity state. The penalties are adequate to ensure that the truth-telling constraints are satisfied. To show $y(\theta^1_1) = \hat{y}(\theta^1_1)$, suppose firm 1 is in its low productivity state but claims to be in its high productivity state. Then for whatever loan $L$ it receives from the bank it must borrow to be able to produce the contracted level of output since $\theta^1_1 q(L) < \theta^2_1 q(L)$ for all $L > 0$. If the firm borrows it is revealed to have misrepresented its productivity state and the penalty $b(\theta^1_2)$ is imposed. If the firm does not borrow and then produces less than the contracted level of output it again is revealed to have misrepresented its state, and the penalty is again imposed. To show $\hat{y}(\theta^2_2) = \hat{y}(\theta^2_2)$, suppose firm 1 claims to be in its low productivity state when it is in its high productivity state. Then for any loan $L$ it receives it will have more than enough funds to produce the contracted level of output. If it produces more than the contracted amount it is revealed to have misrepresented itself and the penalty $b(\theta^1_1)$ is imposed. If it produces the contracted level of output and lends out the excess funds it again is revealed to have misrepresented its productivity state and the penalty is again imposed. However, it is still possible for the firm to claim that it is in its low productivity state when it is actually in its high state. It will do this if for any loan $L$ it can
produce the contracted level of output, earn the contractual profits \( \hat{n}(\theta_1^1) \), and have enough loan funds left over, \( \Delta q^{-1} \), so \( \hat{n}(\theta_1^1) + \Delta q^{-1} \geq \hat{n}(\theta_2^1) \). For the bank to ensure that this action never pays, it must set profits if the firm claims to be in productivity state \( \theta_2^1 \) in excess of \( \hat{n}(\theta_1^1) + \Delta q^{-1} \), where \( \Delta q^{-1} \) is the highest excess loan consistent with the announced productivity of firm 1, and either of the productivities of firm 2. So to ensure the firm tells the truth the bank sets

\[
\hat{n}(\theta_1^1) = \max \{ E[\pi(\theta_1^1)]; \hat{n}(\theta_1^1) + \max \{ \Delta q^{-1}[y^1(\theta_1^1|\theta_1^1)], \Delta q^{-1}[y^1(\theta_1^1|\theta_2^1)] \} \}
\]

this is exactly \( T2T2 \).

Duopolist 1 accepts the contract if

\[
\hat{n}(\theta_1^2) = \hat{n}(\theta_1^1) + \max \{ \Delta q^{-1}[y^1(\theta_1^1|\theta_1^1)], \Delta q^{-1}[y^1(\theta_1^1|\theta_2^1)] \}
\]

since this ensures it more than it could earn in the market, and it will be indifferent between accepting and rejecting the contract if \( \hat{n}(\theta_1^2) = E[\pi(\theta_1^2)] \). Assume that if it is indifferent it chooses the action preferred by the bank. It is the dominant subgame strategy to self-select the contract consistent with its perceived productivity state. The optimal contract for duopolist 2 is designed in the same fashion.

If \( \hat{n}(\theta_2^j) = E[\pi(\theta_2^j)], \ (j = 1, 2) \), the bank can hold the firms to expected duopoly profits. The contracts allow the bank to set loans (output levels) at the unconstrained joint profit maximisation point, and extract all the expected (positive) surplus from control. However, if

\[
\hat{n}(\theta_2^j) = \hat{n}(\theta_1^1) + \max \{ \Delta q^{-1}[y^1(\theta_1^1|\theta_1^1)], \Delta q^{-1}[y^1(\theta_1^1|\theta_2^1)] \}
\]

then the expected surplus the bank is able to extract may not be non-negative. If it is not, the bank will not attempt to control the industry.

References


